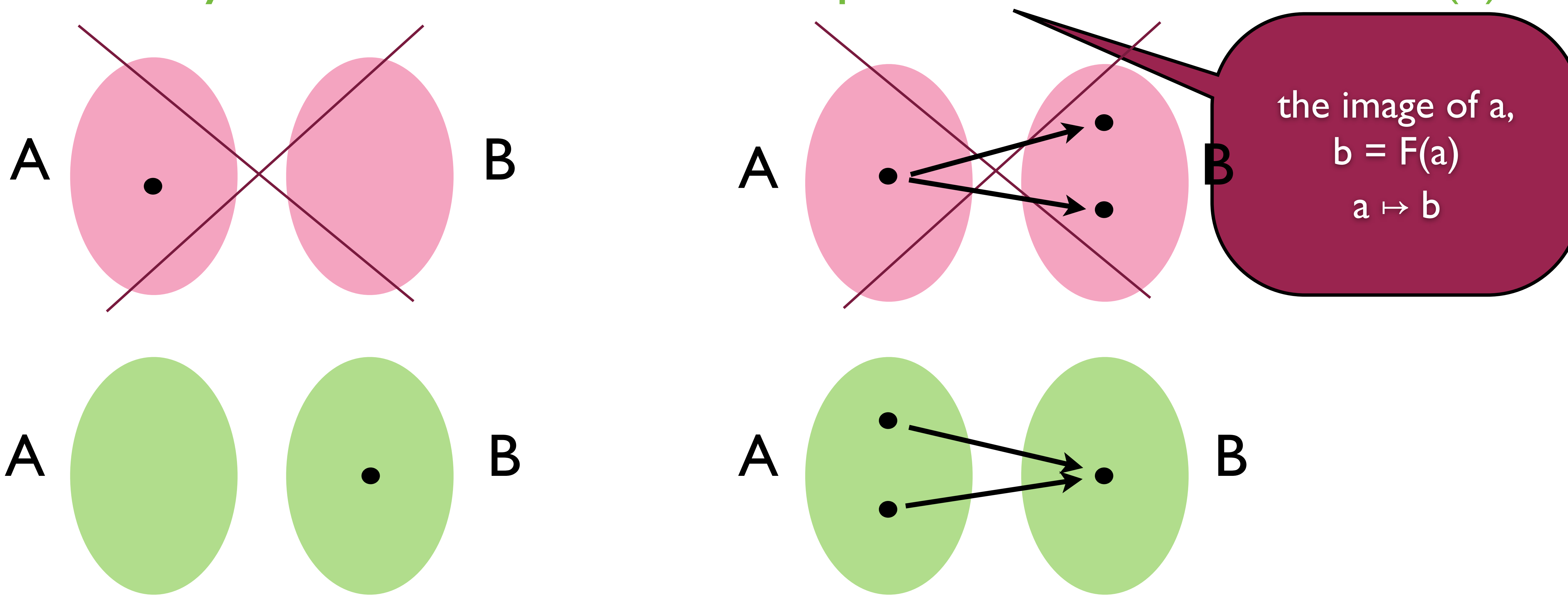
Functions, mappings

Def. If A and B are sets, then F is a function (mapping, Abbildung) from A to B, notation F: A $\longrightarrow$ B iff

for every $a \in A$, there exists a unique $b \in B$ such that $b = F(a)$.



$\{(a, F(a)) \mid a \in A\}$ is the graph of the function F

Functions, mappings

When f: A ⟶ B then dom f = A and cod f = B

domain of F
(Definitionsbereich)

codomain of F
(Wertebereich)

Let f: A ⟶ B and A' ⊆ A.

The image (Bild) of A' is the set f(A') = {f(a) | a ∈ A'} ⊆ B.

f(A') = {b ∈ B | there is an a ∈ A' with b = f(a)}

if a ∈ A', then f(a) ∈ f(A')

So f extends to a function f: $\mathcal{P}$(A) ⟶ $\mathcal{P}$(B), the image-function.

Functions, mappings

Let f: $A \longrightarrow B$ and $B' \subseteq B$.

The inverse image (Urbild) of B' is the set

$$f^{-1}(B') = \{a \mid f(a) \in B'\} \subseteq A.$$

$a \in f^{-1}(B')$ iff $f(a) \in B'$

Again the inverse image induces a function $f^{-1}: \mathcal{P}(B) \longrightarrow \mathcal{P}(A)$, the inverse-image-function.

Lemma F1: Let f: $A \longrightarrow B$, $A' \subseteq A$, and $B' \subseteq B$. Then

$$A' \subseteq f^{-1}(f(A')) \quad \text{and} \quad f(f^{-1}(B')) \subseteq B'$$

(in general no more than this holds)