Formale Systeme

Test 2, Group 1, 2.2.2016

Task 1. (15) Write down the definitions of the following notions:

(a) A relation R is reflexive.

(b) A relation R is transitive.

(c) A function $f\colon A \to B$ is surjective.

Task 2. (20) Let A and B be arbitrary sets and let $A \neq \emptyset$. Prove that a function $f\colon A \to B$ is injective if and only if it has a left inverse, that is, there exists a function $g : B \to A$ such that $g \circ f = \mathrm{id}_A$.

Task 3. (15 + 5 + 10) Let $\Sigma = \{a, b\}$. Let $L = \{a^n b \mid n \in \mathbb{N}\}$.

(a) Prove that $|L| = \aleph_0$.

(b) Write L with a regular expression.

(c) Construct an automaton for L.

Task 4. (20) Let n, m be two natural numbers such that $n|m$. Prove by induction that **for all** $i \in \mathbb{N}$, $n^i|m^i$.

Recall the inductive definition of n^i for a natural number n:

$$n^0 = 1 \text{ and } n^{i+1} = n^i \cdot n.$$

Recall also the definition of divisibility, i.e., $n|m$ iff $m = n \cdot k$ for some natural number k.

Task 5. (15) Construct a DFA for $L = \{w \in \{0,1,2\}^* \mid 3|\,(\#_0(w) + 2\#_1(w))\}$.

Task 6. (15) Let R be a relation on a set X, i.e. $R \subseteq X \times X$. Let $X \neq \emptyset$. Prove that if R is both an equivalence and a partial order, then $R = \Delta_X$.