CONCURRENCY AND SCALABILITY VERSUS FRAGMENTATION AND COMPACTION WITH COMPACT-FIT

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Abstract. We study, formally and experimentally, the trade-off in temporal and spatial overhead when managing contiguous blocks of memory using the explicit, dynamic and real-time heap management system Compact-fit (CF). The key property of CF is that temporal and spatial overhead can be bounded, related, and predicted in constant time through the notion of partial and incremental compaction. Partial compaction determines the maximally tolerated degree of memory fragmentation. Incremental compaction of objects, introduced here, determines the maximal amount of memory involved in any, logically atomic, portion of a compaction operation. We explore CF's potential application space on (1) multiprocessor and multicore systems as well as on (2) memory-constrained uniprocessor systems. For (1), we argue that little or no compaction is likely to avoid the worst case in temporal as well as spatial overhead but also observe that scalability only improves by a constant factor. Scalability can be further improved significantly by reducing overall data sharing through separate instances of Compact-fit. For (2), we observe that incremental compaction can effectively trade-off throughput and memory fragmentation for lower latency.

1. Introduction

Compact-fit (CF) [9] is an explicit, dynamic, and real-time heap management system also known as a memory allocator. Heap management solves the problem of allocating and deallocating memory objects of possibly different size where the order in which the objects are allocated and deallocated may be arbitrary. It is dynamic if allocation and deallocation happens at runtime, as opposed to static, so-called pre-allocation, which may only be done if the amount of memory needed for program execution can be bounded at compile time. It is real-time if the time to allocate, deallocate, and access a memory object is either constant or at most proportional to the size of the object, independent of the overall state of memory and in particular the order in which objects are allocated and deallocated.

Heap management is explicit if deallocation must be invoked by the program using the system. The use of explicit heap management may therefore suffer from the well-known phenomena of memory leaks, where memory objects are continuously allocated but never deallocated, and so-called dangling pointers to memory objects that have been deallocated prematurely and may lead to undefined program behavior when accessed. Compact-fit is not an exception. It does not address the problems of memory leaks and dangling pointers.

Implicit heap management solves the problem of dangling pointers by first determining when to deallocate memory objects safely and then using an underlying explicit heap management to actually deallocate these objects. Garbage collectors are implicit heap management systems which logically operate in two phases that are performed repeatedly. In the first phase allocated but unreachable memory objects are determined, either directly through reference counting and cycle detection, or indirectly through tracing, or some combination of both [2]. A memory object is unreachable if the program has no means of accessing the object, neither directly through some reference nor transitively through other, reachable memory objects. An unreachable memory object may thus be deallocated safely without introducing dangling pointers.

In the second phase unreachable memory objects are deallocated which is done by an underlying memory allocator. In other words, a garbage collector implicitly uses a memory allocator which is, however, often tightly integrated with the garbage collector for performance reasons. Compact-fit is closely related to the integrated memory allocator of the real-time garbage collector Metronome [4].

Garbage collectors solve the problem of dangling pointers but not the problem of memory leaks: a program may continuously allocate memory objects to which it maintains references, e.g. by inserting the objects into an ever-growing hashtable and never removing them again. The result is a so-called reachable memory leak which garbage collectors cannot avoid. In other words, using implicit heap management does not free a program from deallocating memory objects. It only makes deallocation implicit (remove references and return from procedure calls rather than deallocate explicitly) and is therefore safe, i.e., the program still needs to go through the otherwise ever-growing hashtable and remove obsolete data from time to time. Fundamentally, reachable memory leaks are not detected by garbage collectors because reachability is only an overapproximation of memory liveness which itself is undecidable: after some time a memory object may never be accessed again but still remain reachable.

The true power of garbage collection is that it makes the use of the heap compositional. Programs may allocate memory objects, even in imported library code, and pass references to them around without keeping track of when to deallocate the objects as long as the references are eventually removed when the objects are not needed anymore. Compositionality of the heap is key to large-scale program design and particularly useful in concurrent programs where keeping track of when shared memory may be deallocated is especially difficult.

The price to pay for garbage collection is temporal and spatial overhead: computing unreachability (directly or indirectly) is proportional to the size of live, i.e., reachable memory (in the presence of cyclic references which is the case in most non-trivial applications), resulting in lagged deallocation of unreachable memory and thus increased memory consumption. Temporal overhead, when created in so-called stop-the-world fashion, precludes real-time applications. Spatial overhead precludes embedded applications, in particular if deallocation not only lags unreachability but also results in uncontrolled memory fragmentation which may also occur in explicit heap management without any garbage collection.

Temporal and spatial overhead of dynamic heap management, explicit or implicit, cannot be avoided but it can be bounded! The key to enabling dynamic heap management in real-time and embedded applications is to make it incremental and to bound memory fragmentation. Heap management is incremental if it may be done in phases whose durations are constant and which may be interleaved with program execution. The maximum duration of a heap management phase determines the latency introduced by heap management and thus directly defines the compatible class of real-time applications. The drawback of incremental heap management is lower throughput since the sum of the phases of a heap management operation is generally larger than the duration of the operation when not interrupted. Note that the focus of this paper is on making explicit heap management incremental while bounding memory fragmentation, which is one of the two fundamental prerequisites for incremental garbage collection. The other prerequisite is incrementally computing unreachability which is addressed elsewhere, e.g. in Metronome [4].

Memory fragmentation is the phenomenon of unoccupied memory blocks being dispersed in memory (external fragmentation) and/or designated through partitioning (internal fragmentation). If contiguous memory blocks of different size may be allocated and deallocated in arbitrary order, uncontrolled memory fragmentation may lead to unbounded gross memory consumption even if net memory consumption is bounded.

Compact-fit avoids external fragmentation and bounds internal fragmentation through partitioning and so-called partial compaction. Upon deallocating a memory object partial compaction may move another same-size object into its place but only if a given threshold on fragmentation is exceeded. In this case, deallocation takes time linear in the size of the deallocated object. Otherwise, deallocation is constant-time. Memory allocation as well as access are always constant-time. The principle topic of this paper is to make partial compaction incremental such that objects are moved incrementally in phases of constant duration and yet may still be accessed in constant time in between compaction phases. The result is what we call incremental Compact-fit, the first memory allocator that bounds the full spectrum of temporal and spatial overhead of memory allocation, deallocation, and access in terms of configurable constants. With incremental Compact-fit the duration of any heap management activity as well as the degree of memory fragmentation are bounded by constants, which makes this allocator the principle choice for any application in which constant bounds on both temporal and spatial overhead are required.

Note that there are memory allocators that either bound temporal overhead in terms of constants such as Half-fit [28] and TLSF [25] or else spatial overhead such as the allocator of the Jamaica VM [36] but not both. Half-fit and TLSF provide constant-time memory allocation, deallocation, and access, but only control and not bound memory fragmentation through coalescing neighboring, unoccupied memory blocks. The Jamaica allocator avoids external fragmentation and bounds internal fragmentation but at the expense of constant-time memory allocation, deallocation, and access as well as memory locality by allocating small, same-size but generally dispersed memory blocks and assembling them into larger memory objects through trees (logarithmic-time access) or lists (linear-time access) that fit the requested size.

Next, we discuss the design principles and features of Compact-fit before providing an overview of the rest of the paper.

1.1. **Compact-fit.** Compact-fit partitions memory into virtual pages of equal size by maintaining a list of free pages and a segregated list of finitely many so-called size-classes where each size-class is a doubly-linked list of used pages that are further partitioned into virtual, so-called page-blocks of equal and unique size. A memory object is allocated as contiguous block of memory in a free page-block of the size-class with the smallest page-block size that still fits the object. Memory allocation, deallocation, and access takes constant time (unless compaction is necessary when deallocating, which takes linear time in the size of the deallocated object). Allocation of memory objects larger than the page size is not part of CF itself but may be done on top of CF by array, tree-, or list-based data structures that combine sufficiently many pages to accommodate large objects resulting in allocation and deallocation times that are linear and memory access times that are constant, logarithmic, or linear, respectively, in the size of the objects. However, we do not consider large-object management here.

The size-class concept is generally subject to fragmentation through partitioning, that is, to bounded page-block-internal, page-internal, and size-external fragmentation [3], but enables CF to keep memory size-class-compact at all times [9]. Memory is size-class-compact if each of its size-classes is compact. A size-class is compact with respect to a so-called partial compaction bound κ if the size-class contains only non-empty pages of which at most κ are not-full. A size-class is said to be totally compact, fully compact, or partially compact if it is compact with respect to $\kappa = 0$, $\kappa = 1$, or $\kappa > 1$, respectively. Note that, as opposed to the leftover space caused by fragmentation through partitioning, which is wasted for any request, the free space in not-full pages of a size-class, called size-class fragmentation, is wasted for any request but the requests that actually match the size-class. Partial compaction can only control the degree of size-class fragmentation.

CF always keeps all size-classes compact with respect to individual, per-size-class partial compaction bounds $\kappa > 0$. Overall memory fragmentation is therefore bounded and predictable in constant time. Note that $\kappa = \infty$ is also permissible and means that any number of not-full pages in a size-class is tolerated. A memory object is allocated, in constant time, in a free page-block either of a not-full page of the adequate size-class (implicitly compacting allocation), or else, if there is no not-full page in the size-class, of a free page that is then removed from the list of free pages and assigned to the size-class (non-compacting allocation). A memory object is deallocated, either in constant time, by marking the page-block used by the object as free, if the size-class remains partially compact (non-compacting deallocation), or else in linear time in the size of the object, by marking a used page-block of a not-full, so-called source page as free after copying the content of that (source) page-block to the (target) page-block used by the object, which, in this case, must be located in a full, so-called target page (compacting deallocation). If the page in which a page-block was marked as free becomes empty, the page is removed from the size-class and returned to the list of free pages.

In order to facilitate compacting memory that may contain references in time linear in the size of the moved objects, CF maintains a map (A2C) from abstract object addresses that do not change when moving objects, also referred to as handles, to the concrete object addresses in memory. Objects may only refer to other objects using their abstract addresses, which implies that memory access requires one level of indirection, unless compaction is turned off with $\kappa = \infty$. As a result, whenever an object is moved in memory, only its concrete address in the A2C map needs to be updated. CF stores the abstract address of each object in the object itself so that the object's entry in the A2C map can be determined in constant time. Otherwise, determining the abstract addresses of objects selected for compaction, for which only the concrete addresses are known, would require searching the A2C map.

There is also a non-moving version of CF [9], which virtualizes the concrete address space using an additional level of indirection that merely requires reprogramming a map (V2P) from virtual to physical addresses upon compaction instead of moving the actual content of the objects. Since objects do not move, their physical addresses can be used to generate unique abstract addresses, which avoids storing abstract addresses in objects. Nevertheless, in the worst case, the V2P map requires just as much memory as the object storage for abstract addresses. Moreover, experiments have shown that the non-moving version of CF may only pay off when used for larger objects [9]. In the rest of the article, we only consider the moving version of CF.

1.2. **Overview.** After discussing related work (Section 2) and discussing the previously described, moving (and non-incremental) version of CF in detail (Section 3), we first argue probabilistically that, for particular mutator behavior, both compaction and worst-case size-class fragmentation are less likely to happen with increasing partial compaction bounds κ. For systems whose memory resources are less constrained and applications that do not require tight guarantees, partial compaction may therefore be set to large κ, or even turned off entirely. This observation has lead us to develop an optimized, non-compacting version of CF without abstract addressing that does not maintain the A2C map and can therefore be used in any application without modifications. Macrobenchmarks show that the optimized version performs almost as fast as other constant-time state-of-the-art memory allocators. Moreover, less than 5% of the fragmentation can be attributed to size-class fragmentation and the rest to fragmentation through partitioning (Section 8). We argue that partitioning memory as in CF still has the benefit of being subject to a probabilistic and not just an experimental fragmentation analysis (Section 4), at the expense of increased memory consumption.

We then introduce incremental CF for slow systems, at the other end of the spectrum, whose memory resources are constrained and that run applications requiring tight guarantees, in particular on system latency and memory consumption (Section 5). Incremental CF uses a global compaction increment $\iota > 0$, which breaks up compaction into logically atomic operations that do not move more than ι bytes at a time. If n is the degree of concurrency, then there may be at most n pending incremental compaction operations moving objects stored in n source page-blocks from n source pages to n target pages. The memory occupied by the n source page-blocks causes so-called transient size-class fragmentation in the n source pages. The key result is that the time complexity of memory allocation, deallocation, and access remains asymptotically the same as with non-incremental CF while overall memory fragmentation is still bounded and predictable in constant time (Section 6). Incremental CF may improve system latency at the expense of allocation and deallocation throughput and transient size-class fragmentation (Section 8).

Figure 1 gives an intuitive overview of the effect of different versions and configurations of CF on allocation and deallocation throughput, system latency, and memory fragmentation.[1] A configuration 1-CF(κ, ι) denotes a single instance of

[1]See Section 6 for a table with allocation/deallocation complexities of each version.

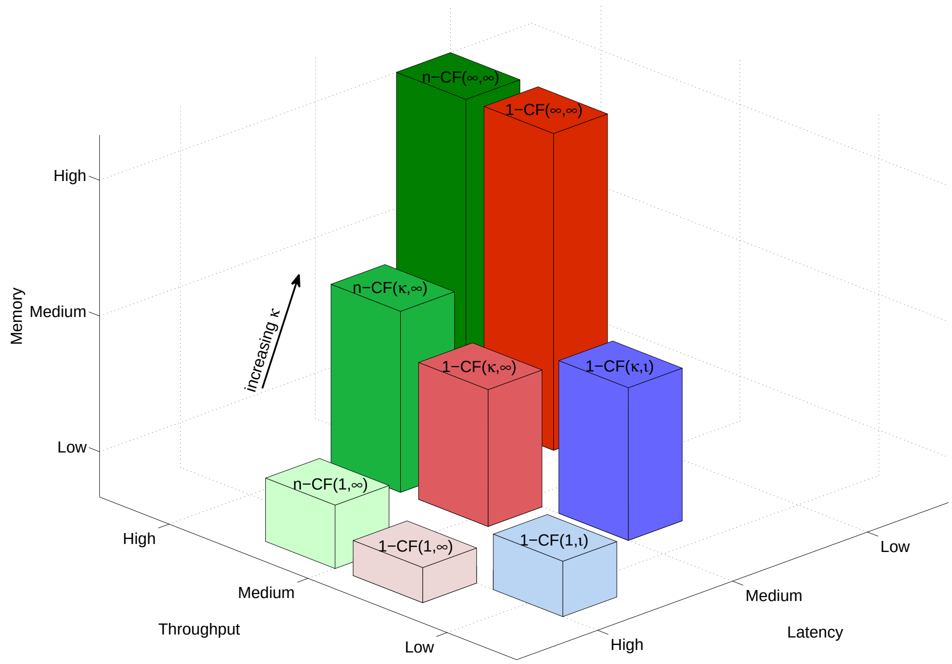


FIGURE 1. Allocation and deallocation throughput, system latency, and memory fragmentation with different versions and configurations of Compact-fit

a CF system with a per-size-class partial compaction bound $\kappa > 0$ and a global compaction increment $\iota > 0$. The instance may be shared by concurrently running threads using a number of different, standard synchronization techniques (Section 7). Incremental compaction is off if $\iota = \infty$. Partial compaction is off if $\kappa = \infty$, which implies that incremental compaction is also off. Full compaction is on if $\kappa = 1$. The fully compacting, non-incremental 1-CF$(1, \infty)$ configuration minimizes memory fragmentation at the expense of throughput and latency. In comparison, the fully compacting, incremental 1-CF$(1, \iota)$ configuration may require more memory because of transient size-class fragmentation and provide less throughput but may reduce latency. With $\kappa > 1$, memory fragmentation may go up proportionally to κ with both configurations while throughput may be higher and latency may be lower as there may be fewer compaction operations. The non-compacting 1-CF(∞, ∞) configuration may provide even higher throughput and lower latency but may also consume even more memory. The key advantage of this configuration is that it may be optimized as mentioned above.

A configuration n-CF(κ, ι) denotes n instances of a CF system, one for each of n threads, which is meant to improve scalability on multiprocessor and multicore systems (Section 7). Compared to the single-instance configurations, throughput may be higher but memory fragmentation may also go up with the compacting configurations since partial compaction bounds are enforced per instance and therefore per thread. Our experiments show that partial compaction on fast systems may only have an effect on scalability by a constant factor since the time required to perform a single compaction operation on such systems is close to the time required to perform any other CF operation, independently of the size of the involved object. More relevant to scalability is the degree of data sharing, in particular, through the A2C map (Section 8).

The contributions of this article are the design, implementation, and comprehensive, formal and experimental evaluation of concurrent versions of (1) an optimized, non-compacting CF system, (2) the previously described, compacting, non-incremental CF system [9], and (3) a new, compacting, incremental CF system.

2. Related Work

Scalability of concurrent memory allocators [7] and garbage collection systems [12, 21] is the key for high performance in parallel environments. We relate our work to dynamic heap management systems of different kinds: explicit sequential allocators, explicit concurrent allocators, and concurrent garbage-collection-based systems with compaction (cf. [17] for an extensive online bibliography).

Most of the established explicit sequential dynamic heap management systems [24, 32] are optimized to offer excellent best-case and average-case response times, but in the worst-case are unbounded in the size of the memory allocation or deallocation request, i.e., depend on the global state of memory. The best known are First-fit, Best-fit [22] and DL [23] with allocation times depending on the global state of memory. Half-fit [28] and TLSF [25] are exceptions offering constant response-time bounds for allocation and deallocation, but even they may suffer from unbounded and unpredictable memory fragmentation.

Several concurrent dynamic memory allocators have been designed for scalable performance on multiprocessor systems. Hoard [7] provides fast and scalable memory allocation and deallocation operations, using locks for synchronization and avoiding false sharing of cache lines. A lock-free memory allocator with lower latency based on the principles of Hoard is given in [26]. A partly lock-free non-portable memory allocator, which requires special operating system support, is discussed in [10]. McRT-Malloc [14] is a non-blocking scalable heap management algorithm, which avoids atomic operations on typical code paths by accessing only thread-local data and uses the same memory layout (pages and size-classes) as CF. None of these systems provides temporal or spatial guarantees.

Incremental compaction typically performs the compaction phase of a garbage collection cycle incrementally, i.e., multiple objects are moved atomically. The incremental compaction algorithms discussed in this paragraph and the following paragraph are based on that concept. Our incremental compaction approach in CF is different. CF allows to move a single object incrementally, which may reduce the latency of a compaction operation even further. There are many concurrent compaction strategies implemented in garbage-collected systems, which do not provide temporal or spatial guarantees. In [11] a parallel stop-the-world memory compaction algorithm is given, where multiple threads compact the whole heap. Compressor [20] is a concurrent, parallel, and incremental compaction algorithm which compacts the whole heap during a single heap pass, achieving perfect compaction. A further parallel incremental compaction approach is presented in [5] where the heap is split into pieces which are compacted one at a time by moving objects to a new memory region. A fixup pass takes care of reference updates. An algorithm with improved compaction pause times via concurrent reference updates, using only half of the heap, is given in [29]. Each thread performs reference updates proportional to its allocation requests. In [18] the authors discuss object replication versus forwarding pointer based compaction strategies. They evaluate the performance in a non-concurrent virtual machine and show that object replication may provide higher throughput there.

Garbage-collecting heap management systems that do provide response-time guarantees on allocation and deallocation operations are Jamaica [36] as well as Metronome [4]. With Jamaica allocation and deallocation take linear time in the size of the operation request. Compaction is not needed since memory objects do not occupy contiguous blocks of memory. Another garbage collection approach based on non-contiguous memory allocation is discussed in [30] where memory access can be performed in constant time. Metronome is a time-triggered garbage collector, which uses the same memory layout as CF. Compaction in Metronome is part of the garbage collection cycles. The time used for compaction is estimated to at most 6% of the collection time [3], without precise guarantees. The performance of Metronome depends highly on the mutator behavior. MC^2 [34] is an incremental soft real-time garbage collector designed for memory constrained devices, which cannot provide hard guarantees on maximum pause time and CPU utilization, but comes with low space overhead and tight space bounds. Stopless [31] is another garbage collector with soft guarantees on response times. It provides low latency while preserving lock-freedom, supporting atomic operations, controlling fragmentation by compaction, and supporting multiprocessor platforms. The main contribution of Stopless is a compaction algorithm which moves objects in the heap concurrently with program execution. Exact bounds for response times, as well as fragmentation, are missing in Stopless. Another incrementally compacting real-time garbage collection algorithm where memory is divided into multiple pieces of equal size, which get scavenged periodically resulting in bounded pause times is presented in [27]. In [19] the authors show experimentally that the cost of handles in a real-time garbage collector is negligible in comparison to implementations that do not use handles. In [6] the authors discuss worst-case fragmentation bounds for different heap management strategies. Scheduling of garbage collection tasks in real-time environments is discussed in [35].

We remark that CF, like many of the above mentioned systems, is based on segregated lists. Approaches that are not based on segregated lists, but rather on data structures which maintain locality of objects, are known to perform better when accessing objects by utilizing memory caches more effectively. However, the use of segregated lists enables providing and trading-off temporal and spatial guarantees.

3. Non-incremental Compact-fit

Compact-Fit (CF) is an explicit, dynamic heap management system that provides strict temporal and spatial (fragmentation) guarantees. Allocation as well as deallocation without compaction takes constant time, whereas deallocation with compaction takes linear time in the size of the object.

To be precise, there are two CF implementations [9], but in this article we only focus on the more fundamental so-called moving implementation.

The set-up of CF is as follows: The memory is divided in pages of equal size. Each page (in use) contains a certain number of constant-sized page-blocks. In total

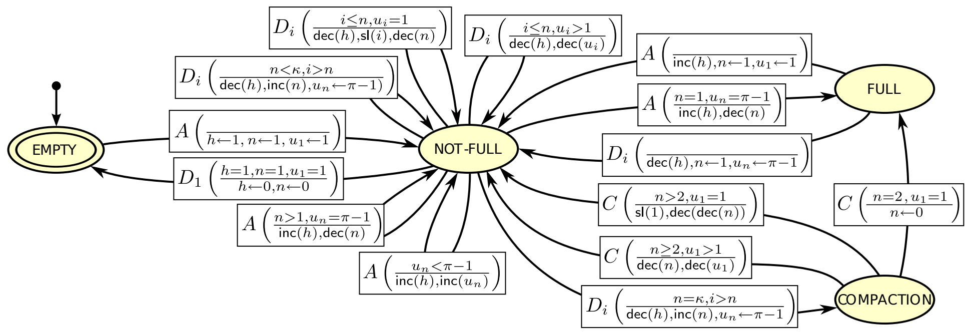


FIGURE 2. Size-class automaton with $\pi > 1$

there are finitely many available page-block sizes, which determine to which size-class a page belongs (namely all pages with a given page-block size belong to one size-class). The pages are assigned to a size-class only if they are used (non-empty). The number of page-blocks π per page in a size-class is therefore determined by the size of a page and the block size. The state of a size-class depends on the state of the pages that belong to it and is described by the values of the variable tuple

$$\langle h, n, u_1, \dots u_n \rangle$$

where h is the total number of allocated page-blocks in the size-class (its portion of the heap), n is the number of not-full pages, and for each not-full page i, u_i is the number of used page-blocks in the page.

An allocation request for an object of size l is served by a page of a best-fitting size-class. That is, for allocating an object a single page-block is used in a page whose page-blocks are of the smallest size still big enough to fit l. For example, if there are two size-classes, one with page-blocks of size 10 and one with page-blocks of size 20 units, then an allocation request for an object of size $l \in \{11, 12, \dots 20\}$ will be served by a page of the size-class 20. If all pages in the best-fitting size-class are full, then a new empty page is added to the size-class and the object is allocated in this new page.

We allow for a constant number $\kappa > 0$ of not-full pages per size-class. The aim in the design of CF is to control size-class fragmentation, which is the space occupied by free page-blocks in not-full pages (space not available for allocation in other size-classes). If deallocation happens, and the number of not-full pages becomes $\kappa + 1$ after this deallocation operation, then compaction is invoked. Compaction consists of moving a single object from a not-full page to the page-block of the deallocated object, which is the only empty page-block in that page. As a result, after compaction, the number of not-full pages in a size-class does not exceed κ.

An object is assigned a unique abstract address (handle), which has to be dereferenced whenever accessing an object field. This introduces a constant object dereferencing overhead but facilitates predictability of reference updates during compaction, i.e., whenever an object is moved in memory it requires to update just its abstract address space entry.

We show the CF algorithm in full detail in Figure 2, using a deterministic automaton, one per size-class. For presentation purposes, we draw a quotient of the state space of the size-class: EMPTY stands for the single state $\langle 0, 0 \rangle$ representing an empty size-class; NOT-FULL represents all states with at least one not-full page where no compaction is needed, that is $\langle h, n, u_1, \dots u_n \rangle$ with $0 < n \leq \kappa$; the state FULL represents all states with no not-full pages and at least one full page, that is $\langle h, 0 \rangle$ with $h > 0$; finally, COMPACTION represents states $\langle h, \kappa+1, u_1, \dots, u_{\kappa+1} \rangle$ in which compaction must be invoked.

The transitions in the automaton are labelled in the following way: A denotes allocation, D_i deallocation in page i (which may be full or not-full, the latter is recognized by $i \leq n$), and C denotes a compaction step. Moreover, a transition fires if its premise is satisfied, and results in a change of state described by its conclusion. For updating a state, we use the operators $\leftarrow$ for assignment, dec for decrement, inc for increment, and sl for shift left. More precisely, $\mathsf{sl}(i)$ removes u_i from a state sequence, i.e., it changes a state $\langle h, n, u_1, \dots, u_{i-1}, u_i, u_{i+1}, \dots, u_n \rangle$ to the sequence $\langle h, n, u_1, \dots, u_{i-1}, u_{i+1}, \dots, u_n \rangle$.

We explain several instructive transitions in full detail, and refer the reader to Figure 2 for the full algorithm.

$A\left(\frac{}{h\leftarrow 1,\, n\leftarrow 1,\, u_1\leftarrow 1}\right)$ from EMPTY to NOT-FULL

This transition fires whenever allocation is requested in the empty state. As a result the state changes to $\langle 1, 1, 1 \rangle$.

$D_i\left(\frac{i \leq n,\, u_i > 1}{\mathsf{dec}(h),\, \mathsf{dec}(u_i)}\right)$ from NOT-FULL to NOT-FULL

This transition is taken upon a deallocation step in a not-full page which remains non-empty after the deallocation. The change in the state is that the number of used page-blocks is decremented by 1, and, as in every deallocation step, the heap size decreases by 1.

$A\left(\frac{}{\mathsf{inc}(h),\, n\leftarrow 1,\, u_1\leftarrow 1}\right)$ from FULL to NOT-FULL

Whenever an object is allocated in a state of the class FULL a new empty page has to be added to the size-class, and allocation happens in this page. As a result this new page becomes the only not-full page of the size-class with a single page-block used. The value of h increases by one, as with any allocation operation.

$D_i\left(\frac{n=\kappa,\, i>n}{\mathsf{dec}(h), \mathsf{inc}(n), u_n\leftarrow \pi-1}\right)$ from NOT-FULL to COMPACTION

With this transition we are in a situation when after the required deallocation operation, in the i-th page which was full, we have more than κ not-full pages. Therefore, compaction must be invoked in the next step.

$C\left(\frac{u_1=1,\, n>2}{\mathsf{sl}(1),\, \mathsf{dec}(\mathsf{dec}(n))}\right)$ from COMPACTION to NOT-FULL

Being in state COMPACTION, the next transition has to be of type C. Moreover, note that $n = \kappa + 1 \geq 2$. During the compaction step a page-block is moved from the first not-full page (represented by u_1) to the last not-full page, namely the one in which deallocation just happened. This particular transition fires if the first not-full page has just one page-block. As a result it becomes empty after the transition,

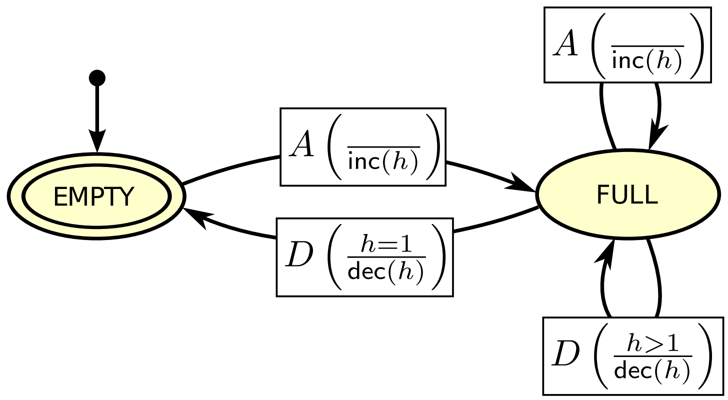


FIGURE 3. Size-class automaton with $\pi = 1$

whereas the last not-full page becomes full. Since $n > 2$ the transition leads to the state NOT-FULL. The operation shift left is needed to remove the value u_1 for the now empty page.

We note that in case $\pi = 1$, i.e., in a size-class in which each page consists of exactly one page-block, there are no not-full pages. A page is either empty or full. In this case compaction can never happen. Therefore, the size-class automaton simplifies significantly as shown in Figure 3.

We have chosen the automaton presentation of CF in order to prepare the ground for the concurrent version. For the original presentation of CF, we refer the interested reader to [9]. We extend the non-incremental CF with blocking and non-blocking synchronization mechanisms so that multiple threads can share a single (or multiple) instance(s). In particular, we make the size-class automaton transitions (including a combination of a deallocating transition followed by a compacting step) atomic. As a result, multiple threads can execute and use CF in parallel, interleaving between the atomic transitions. The details of the particular implementation and the various choices of synchronization mechanisms are discussed in Section 7. The results are encouraging for throughput oriented environments, see Section 8.

4. Probabilistic Analysis

We present a probabilistic analysis of CF which shows that, for particular mutator behavior, both compaction and worst-case size-class fragmentation are less likely to happen with increasing partial compaction bounds κ. Compaction may therefore be set to large κ or even turned off if guarantees on memory fragmentation are not required.

We conjecture that compaction in CF may actually be turned off in some applications while maintaining bounded memory fragmentation with high probability. The probabilistic analysis of CF we present here is not a complete proof of this conjecture but nevertheless motivates non-compacting CF and points in the direction of potential solutions outside the scope of this paper which will need to involve representative classes of mutator behavior. Interestingly, it is the partitioned memory layout of CF that allows for such an analysis, since the partitioning into pages and size-classes significantly reduces the state space of the model. Other memory allocators may not allow such an analysis.

We aim at answering the following two questions:

1. What is the probability that compaction happens?
2. What is the probability of worst-case fragmentation?

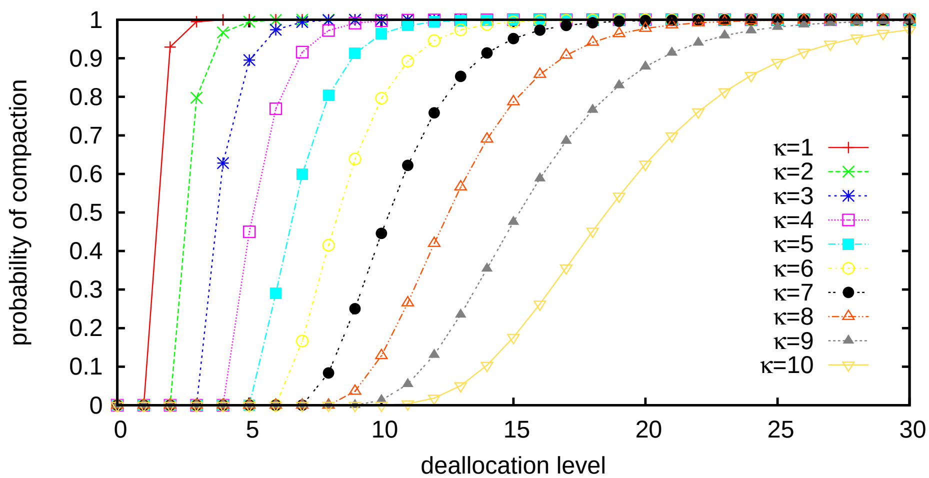
We analyze the behavior of CF given a mutator, which is a sequence of allocations A and deallocations D, hence a word in $\{A, D\}^*$. A mutator is not aware of the internal CF configuration, e.g. in which page deallocation happens. Therefore, we abstract away from the index i in the deallocation label D_i and the CF size-class automaton becomes a probabilistic I/O automaton (PIOA)[2] [37], with input actions A and D provided by the mutator, and an output action C provided by CF. The states of this automaton are either input states in which A and D are enabled, or output states in which C is enforced, which makes it simpler than general PIOA. In an input state, each input action leads to a discrete probability distribution over possible next states. Hence, in an input state, if A happens, we reach a next state with a given probability, and the sum of the probabilities after A equals 1. Symmetrically, after D we reach a next state with a given probability and the sum of the probabilities after D equals 1. In an output state, the single output action C happens with probability 1. For brevity we only discuss in detail the behavior of a single state. In a state $\langle h, n, u_1, \ldots, u_n\rangle$ with $n \leq \kappa$, upon deallocation D, there are several possible next states that are reached with different probabilities: for all i with $u_i > 1$, with probability $\frac{u_i}{h}$ deallocation happens in the not-full page i which will remain not-full afterwards and the next state becomes $\langle h-1, n, u_1, \ldots, u_{i-1}, u_i-1, u_{i+1}, \ldots, u_n\rangle$; for all i such that $u_i = 1$ with probability $\frac{1}{h}$ deallocation happens in page i reducing the number of not-full pages and the next state is $\langle h-1, n-1, u_1, \ldots, u_{i-1}, u_{i+1} \ldots, u_n\rangle$; and with probability $\frac{h-\sum_i u_i}{h}$ the next state is $\langle h-1, n+1, u_1, \ldots, u_n, \pi-1\rangle$ as deallocation happens in a full page. The allocation and compaction transitions remain the same as in the deterministic automaton, they happen with probability 1 in states in which they are enabled. This way we get the *full* PIOA model, with initial state $\langle 0, 0\rangle$.

The full PIOA model together with a mutator induces a discrete-time Markov chain, the *full* DTMC, by pruning out the allocation/deallocation possibilities that the mutator does not prescribe in each state and abstracting away from the transition labels. The full DTMC model results in a large state space already for small values of h, π, and κ.

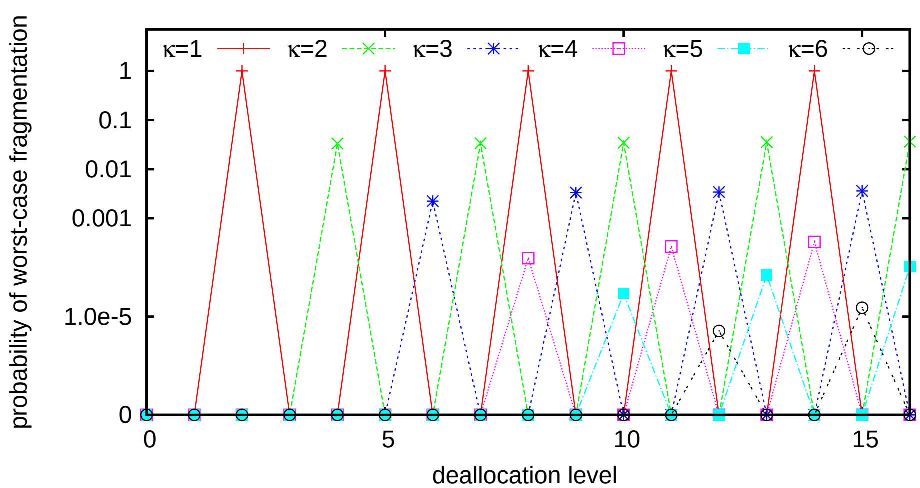
To reduce the number of states, we consider only mutators of the shape $A^h D^d$ which perform h allocations followed by d deallocations. We analyze portions of the full model by setting the state reached after performing h allocations as initial state. This is the state $\langle h, 0\rangle$ if $h \mod \pi = 0$, or $\langle h, 1, h \mod \pi\rangle$ otherwise. Then we consider the portion of the full model reachable in d deallocations. We refer to d as the *deallocation level*. Even such versions of the full model are too big: for $h = 80$, $\pi = 10$, and $\kappa = 5$ the DTMC model in Prism [1] has 1429506 states and 2818395 transitions, and for $h = 80$, $\pi = 10$, and $\kappa = 6$, Prism runs out of memory.

The probability of compaction is the probability of reaching a compacting state, i.e., a state with $n = \kappa + 1$. The probability of reaching a specified state in a

[2]The full definition of a PIOA is out of scope of this paper, instead of giving the general definition we describe the concrete CF size-class automaton as a PIOA.



(a) compaction



(b) worst-case fragmentation

FIGURE 4. Probability of reaching compaction and worst-case fragmentation

DTMC is the sum over all paths of the probability of reaching the state along a path, where the probability of reaching the state along a path is calculated as the product of the probabilities on the path until the state is reached, or equals zero if the state is not reached (in at most d steps). We run both Prism and our own program for exact calculation of the probability of compaction on the model (in Prism only as long as no state space explosion occurs).[3] The results of our exact calculations and of Prism coincide, and they are presented in Figure 4(a) for the

[3]Prism is a general multi-purpose probabilistic model checker applicable to many different models. As all model checkers it suffers from state space explosion. We have implemented a simple single-purpose program that calculates the probabilities of the full DTMC as described

values $h = 1400$, $\pi = 100$, and varying values of κ and d. The particular values of h and π are not significant, we have chosen them so that the probability graphs are sufficiently apart from each other. As expected, the probability of reaching a state where compaction happens for a fixed κ increases with increasing d, and it overall decreases when increasing κ.

Given a state $\langle h, n, u_1, \dots, u_n \rangle$ the (size-class) fragmentation in this state is calculated as $F = n \cdot \pi - \sum_{i=1}^{n} u_i$. The probability of worst-case fragmentation is the probability of reaching a worst-case fragmentation state, i.e., a state with fragmentation $F = \kappa \cdot (\pi - 1)$. The results are shown in Figure 4(b) for $h = 120$, $\pi = 3$, and varying values of κ and d. We present the results for small values of π so that the effect of emptying a page within d deallocations can be seen even for small values of d. Given a partial compaction bound κ, the probability of worst-case fragmentation oscillates periodically as d increases, reaching a maximum value for certain values of d. This maximal probability of worst-case fragmentation decreases with increasing κ, as intuitively expected. Note that the y-axes has a logarithmic scale, and the maximum probabilities of worst-case fragmentation are very low, for $\kappa > 1$.

5. Incremental Compact-fit

For applications which require low latency and run on memory-constrained systems, we provide an extension of CF that allows for incremental compaction, i.e., incremental moving of a single object.

The incremental extension of CF performs compaction, i.e., moving of a single object, by an incremental moving operation. The reason why compaction is made incremental is its dominating linear complexity. This incremental extension is the first step towards a design of latency-efficient concurrent CF. For a concurrent incremental version of CF, allocation, deallocation, and incremental compaction are made atomic, leaving space for other interleaving threads between the atomic steps. As a result the waiting times of concurrent threads, and therefore their response times, decrease, although the compaction throughput may also decrease.

There is a global fixed compaction increment $\iota > 0$ which determines the portion of a page-block being moved in an incremental step. The value of ι may even be larger than some page-block sizes, in which case the whole compaction operation is done non-incrementally, in one step. We refer to a page-block under incremental moving as the *source page-block*, and the page-block to which the object is moved as the *target page-block*. The state of each size-class and its administration gain complexity in the incremental version. In a size-class, apart from the full and not-full pages, there may exist one *source page*. In a source page there are used page-blocks and source page-blocks. The latter are page-blocks that are in the process of being incrementally moved. One source page suffices, since compaction in CF requires moving a used page-block which is now always taken from the source page. Allocation never happens in a source page. A source page always contains at least one used page-block. If a source page looses all its used page-blocks (due to deallocation or compaction), it is removed from the size-class and placed into a global pool E of *emptying source pages*. All pages in the pool contain page-blocks that are involved in ongoing incremental compaction operations. The

above. The results coincide with Prism, but with our simple program we were able to calculate the probabilities on our simple models for larger models than with Prism.

space occupied by source page-blocks and free page-blocks in (emptying) source pages, which is (temporarily) not available for allocation in any size-class, is called transient size-class fragmentation. When all incremental compaction operations in an emptying source page finish, then the page is returned to the global list of free pages. On the other hand, if all incremental compaction operations within a source page finish, i.e., the source page has no more source page-blocks, and if there are still used page-blocks in the source page, then there are two possibilities: (1) the source page becomes a not-full page, if the number of not-full pages is smaller than the partial compaction bound, or (2) the source page is kept as a potential source page without source page-blocks, otherwise. The evolution of a page is shown in Figure 5.

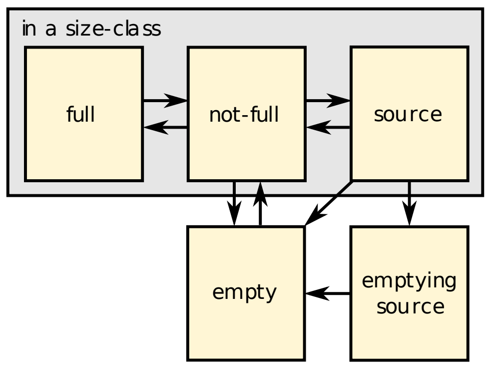


FIGURE 5. The lifetime of a page

The state of a size-class is described by a tuple

$$\langle h, n, u_1, \ldots, u_n, u_s, s, m_1, \ldots, m_s \rangle$$

where, as before, h denotes the current heap size, n is the number of not-full pages such that $n \leq \kappa + 1$ with κ being the partial compaction bound, and the values of $u_1, \ldots, u_n$ are the numbers of used page-blocks in the not-full pages, respectively. The value of u_s equals the number of used page-blocks in the source page, with $u_s = 0$ representing that there is no source page in the size-class. The variable s contains the number of source page-blocks in the source page and equals 0 if there is no source page. Note that $s = 0$ and $u_s > 0$ represents the existence of a potential source page, as discussed above. Finally, $m_1, \ldots, m_s$ are the sizes of the portions of the s source page-blocks that have already been moved.

Figure 6 shows an abstraction of the size-class behavior. Similar to Figure 2, we use abstract states to describe the state changes: EMPTY stands for the single state $\langle 0, 0, 0, 0 \rangle$ representing an empty size-class; the state NOT-FULL, no source represents all states with at least one not-full page where no compaction is needed and no source page is present, that is $\langle h, n, u_1, \ldots u_n, 0, 0 \rangle$ with $0 < n \leq \kappa$; the state FULL, no source represents all states with no not-full pages, at least one full page, and no source page, that is $\langle h, 0, 0, 0 \rangle$ with $h > 0$; NOT-FULL, source represents all states with at least one not-full page where no compaction is needed and a source page, that is $\langle h, n, u_1, \ldots u_n, u_s, s, m_1, \ldots, m_s \rangle$ with $0 < n \leq \kappa$, $u_s > 0$; FULL, source represents all states with no not-full pages, at least one full page, and a source page, that

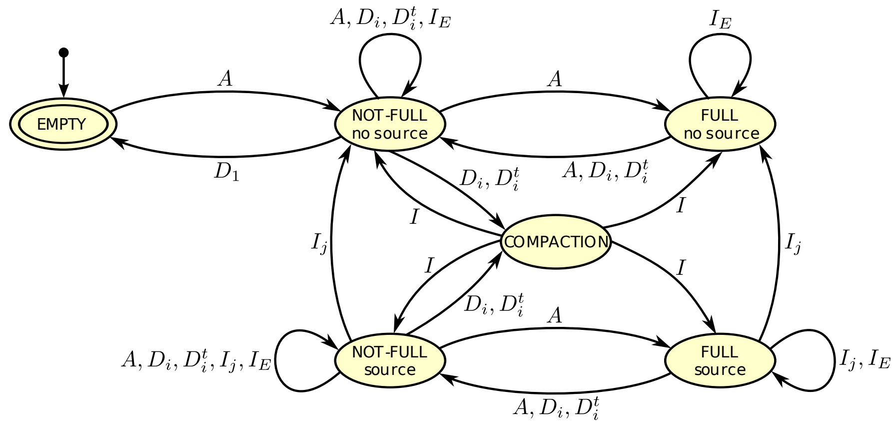


FIGURE 6. Incremental size-class automaton with $\pi > 1$

is $\langle h, 0, u_s, s, m_1, \ldots, m_s\rangle$ with $h > 0$ and $u_s > 0$; finally, COMPACTION is used to represent states $\langle h, \kappa+1, u_1, \ldots, u_{\kappa+1}, u_s, s, m_1, \ldots, m_s\rangle$ in which compaction must be invoked. We note that the automaton and the discussion in this section is under the assumption that the number of page-blocks in a page is larger than 1, $\pi > 1$. The degenerate case with $\pi = 1$ is of no interest.

A state change in a size-class happens upon allocation (A), deallocation (D_i, D_i^t), or incremental compaction $(I,\ I_j,\ I_E)$ transitions. A transition I represents an initial incremental compaction step, I_j is any further incremental compaction step which involves a source page, and I_E is a further incremental compaction step which involves an emptying source page.

We next present the actual changes of states in a size-class in full detail upon allocation, deallocation, and incremental compaction.

Allocation. Allocation steps are the same as in the non-incremental automaton since the source page is not influenced by allocation. In detail, in a state $\langle h, n, u_1, \ldots, u_n, u_s, s, m_1, \ldots, m_s\rangle$ there are three cases:

1. If $n = 0$, that is, there are no not-full pages, then after allocation h increases by 1, n becomes 1, and u_1 becomes 1.
2. If $0 < n \leq \kappa$ and $u_n < \pi - 1$, that is, there is a not-full page and after an allocation it will not get full, then both h and u_n increase by 1.
3. If $0 < n \leq \kappa$ and $u_n = \pi - 1$, that is, a not-full page will get full, then h increases by 1 and n decreases by 1. Note that this may change a state from "not-full" to "full" in case $n = 1$.

Allocation is not possible in a "compaction" state, i.e., a state with $n = \kappa + 1$.

Deallocation. We distinguish two types of deallocation steps denoted by D_i and D_i^t. A step D_i denotes deallocation in page i where the deallocated page-block is not a target of an ongoing incremental moving. In contrast, D_i^t denotes deallocation in page i of a page-block which happens to be a target of an ongoing incremental moving. If $i = 0$, then deallocation happens in the source page; if $1 \leq i \leq n$, then deallocation happens in one of the not-full pages; and if $i > n$ a page-block is deallocated in a full page.

Similar to the non-incremental CF, the change of state after D_i can be described by the following cases:

1. If $1 \leq i \leq n \leq \kappa$ and $u_i > 1$, or if $i = 0$, $u_s > 1$, and $n \leq \kappa$, i.e., deallocation happens in a not-full or source page which will not get empty(ing), then h decreases by 1 and either u_i or u_s decreases by 1, respectively.
2. If $1 \leq i \leq n \leq \kappa$ and $u_i = 1$, i.e., deallocation happens in a not-full page which becomes empty afterwards, then both h and n decrease by 1, and the variable u_i is removed from the state.
3. If $i = 0$, $u_s = 1$, and $n \leq \kappa$, i.e., deallocation happens in a source page which becomes emptying afterwards, then the source page is moved to the pool of emptying source pages E and both s and u_s are set to 0. As a result the size-class does not have a source page.
4. If $i > n \leq \kappa$, which means that deallocation happens in a full page, then h decreases by 1, n increases by 1, and u_n gets the value $\pi - 1$. If originally $n = \kappa$, then this step triggers a compaction operation.

In addition, there are four cases describing the change of state after D_i^t steps. They correspond to the cases for D_i except that at the end of such a step the ongoing incremental compaction operation to the deallocated target page-block is canceled, the target page-block is deallocated, and the source page-block is deallocated. Hence, the (canceled) ongoing compaction operation finishes earlier than it normally would. We refer to the situation when a thread performs a D_i^t step as a *deallocation conflict*.

Deallocation is also not possible in a “compaction” state with $n = \kappa + 1$.

Incremental compaction. Incremental compaction is triggered in case $n = \kappa+1$, just like compaction is triggered in the non-incremental CF. In addition, there may be incremental compaction steps involving emptying source pages from any other state, and incremental compaction steps involving the source page from any state with a source page.

In a state $\langle h, \kappa + 1, u_1 \dots, u_{\kappa+1}, u_s, s, m_1, \dots, m_s\rangle$ an initial incremental compaction step is the only possible step. Note that in such a state $u_{\kappa+1} = \pi - 1$ since the previous step was a deallocation in a full page. We refer to this unique free page-block in the last not-full page as `tb`. The initial incremental compaction step must be atomic together with the preceding deallocation step. We use β to denote the size of page-blocks in the size-class. We have the following cases:

1. If $u_s = 0$, meaning that there is no source page in the size-class, then since $n = \kappa + 1 \geq 2$ the first page becomes the new (potential) source page, i.e., u_s is assigned the value of u_1, s becomes 0, n decreases by 1, and u_1 is removed from the state. After this, the state is no longer a “compaction” state.
2. If $u_s > 0$, then a source page-block `pb` is to be moved to `tb`. There are two possible cases:
   - The page-block `pb` is not a target page-block of an ongoing incremental moving operation. In this case there are two subcases representing an initial incremental compaction step: (1) if $\iota < \beta$, in which case the compaction operation needs more than just one step, then u_s decreases by 1, s increases by one, m_s

is assigned the value of ι and a portion of size ι is moved from pb to tb; (2) if $\iota \geq \beta$, then the whole pb is moved to tb in one step and u_s decreases by 1.

- The page-block pb is a target of a (unique) ongoing incremental operation from a source page-block sb. In this case we are in a situation of a *compaction conflict*. Note that sb must be in an emptying source page in E. Then the ongoing incremental moving operation from sb to pb is canceled, pb is deallocated, and a new initial incremental moving operation starts from sb to tb. Again u_s decreases by 1.

In any case, n decreases by 1. In case $u_s = 0$, the source page becomes emptying, it is moved to the pool of emptying source pages E, and s becomes 0.

Note that the chosen way to resolve the compaction conflict is crucial for bounded compaction response times, since it avoids transitive compaction chains. Namely, a compaction conflict ends an existing compaction and starts a new one, so the duration of a particular compaction operation may only decrease due to a compaction conflict.

In addition, there are three more cases for a change of state due to an ongoing incremental compaction step I_j, where j is an index of a source page-block in the source page that the incremental compaction step applies to. In a state $\langle h, n, u_1 \ldots, u_n, u_s, s, m_1, \ldots, m_s \rangle$ where I_j is applicable, i.e., $u_s > 0$ and $s \geq j$, after an incremental compaction step I_j we have:

3. If $m_j + \iota < \beta$, then m_j is incremented by ι, i.e., another portion of the source page-block gets copied to the target page-block.
4. If $m_j + \iota \geq \beta$ and $s > 1$, i.e., this is the last incremental step for the compaction operation which still keeps the source page, then the number of source page-blocks s decreases by 1, the variable m_j is removed from the state.
5. If $m_j + \iota \geq \beta$ and $s = 1$, i.e., the compaction operation finishes after this incremental step and the source page will no longer exist in the size-class, then s gets the value 0. Furthermore, the source page either becomes a not-full page if $n < \kappa$ (in which case n increases by 1, u_n is assigned the value of u_s, u_s becomes 0) or it is kept as a potential source page.

Finally, there is a possibility for incremental operations I_E which do not change the state, but only change the global pool E of emptying source pages. We skip the details on the description and the update of E due to I_E operations.

We remark that the behavior of any thread can be expressed by a sequence of allocations and deallocations. If a deallocation triggers compaction, then before the thread can continue with any other allocation or deallocation operation all incremental steps needed for the compaction must be finished. The first of these steps is an initial incremental compaction step I which may be an initial incremental moving step in case of compaction conflict. If it is the case, then all other incremental steps are of type I_E. Otherwise, if there is no compaction conflict, a sequence of I_j incremental steps will be performed, and in case the source page becomes emptying a sequence of I_E incremental steps, in order to complete the compaction operation.

6. Complexity vs. Fragmentation

Table 1 shows the time complexity of malloc and free as well as the worst-case system latency, memory size, and size-class fragmentation per CF configuration with n threads and m per-thread-allocated page-blocks in a size-class with π page-blocks of size β per page. The fragmentation caused by partitioning memory [3, 9]

	malloc	free	latency	memory size	size-class fragmentation
1-CF(∞,∞)	$O(n)$	$O(n)$	$O(1)$	$O(n*m*\pi*\beta)$	$O(n*m*(\pi-1)*\beta)$
1-CF(κ,∞)	$O(n)$	$O(n+\beta)$	$O(\beta)$	$O((n*m+\kappa*(\pi-1))*\beta)$	$O(\kappa*(\pi-1)*\beta)$
n-CF(∞,∞)	$O(1)$	$O(1)$	$O(1)$	$O(n*m*\pi*\beta)$	$O(n*m*(\pi-1)*\beta)$
n-CF(κ,∞)	$O(1)$	$O(\beta)$	$O(\beta)$	$O(n*(m+\kappa*(\pi-1))*\beta)$	$O(n*\kappa*(\pi-1)*\beta)$
1-CF(κ,ι)	$O(n)$	$O(n+\beta+\lfloor\frac{\beta}{\iota}\rfloor)$	$O(\min(\beta,\iota))$	$O((n*m+n*\pi+\kappa*(\pi-1))*\beta)$	$O((n*\pi+\kappa*(\pi-1))*\beta)$

TABLE 1. Time complexity of malloc and free as well as worst-case system latency, memory size, and size-class fragmentation per CF configuration and size-class

is not considered here. Although the partial compaction bound κ and the compaction increment ι are kept constant in our current implementations, both κ and ι may be changed dynamically at runtime, which is an interesting topic for future work. System latency is here the portion of the delay a thread may experience, from invoking malloc or free until the operation actually begins executing, caused by currently executing, non-preemptive CF operations, not including the synchronization overhead. Recall that i-CF(κ,ι) denotes a CF configuration where i instances of concurrent CF run in parallel with partial compaction bound κ and compaction increment ι. If $\iota=\infty$, then incremental compaction is turned off. If $\kappa=\infty$, then compaction (and hence also incremental compaction) is turned off.

Since all operations of the non-compacting 1-CF(∞,∞) configuration take constant time, the complexity of malloc and free only depends linearly on the number of competing threads assuming fair scheduling. System latency is bounded by a constant. However, the worst case in memory consumption is one page for each allocated object due to potentially high size-class fragmentation, which has asymptotically the same bound as the overall memory consumption. The compacting 1-CF(κ,∞) configuration trades-off complexity of free and worst-case latency for better bounds on memory consumption by limiting size-class fragmentation through partial compaction. Note that in this case size-class fragmentation is independent from the number of threads and allocated objects.

The results for the n-CF configurations, in particular the worst cases in memory size and size-class fragmentation, as shown here, are obtained under the assumption that there is no sharing among the n CF instances. The time complexity of malloc and free of both multiple-instance configurations goes up to the respective single-instance cases if there is sharing among the n CF instances. While the non-compacting n-CF(∞,∞) configuration requires in the worst case no more memory than the non-compacting single-instance configuration, the compacting n-CF(κ,∞) configuration actually does require in the worst case more memory than

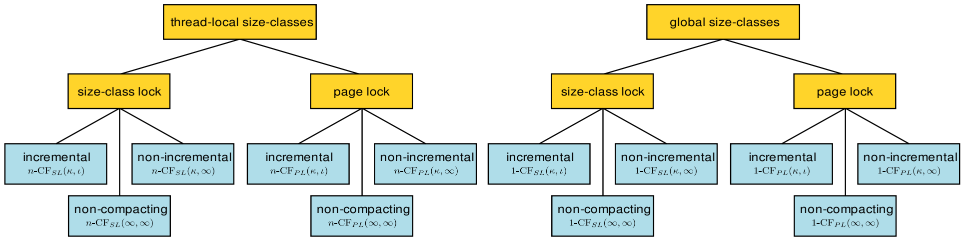


FIGURE 7. Concurrent CF versions

the compacting single-instance configuration since partial compaction is performed per instance. However, allocation and deallocation throughput may increase with both multiple-instance configurations with a decreasing degree of sharing among the n CF instances (without an increase in worst-case system latency).

The incremental 1-CF(κ, ι) configuration actually improves the worst case in system latency whenever the compaction increment ι is less than the page-block size of the size-class with the largest page-blocks, at the expense of the complexity of free through more preemptions and at the expense of memory consumption through increased transient size-class fragmentation. In comparison to the non-incremental, compacting 1-CF(κ, ∞) configuration, there may be up to n additional (emptying) source pages in the system where n is the number of threads. The worst case in non-transient size-class fragmentation does not increase.

7. Implementation

Sequential CF [9] uses three data structures to manage its heap: abstract address, page, and size-class. Additionally, empty pages and available abstract addresses are organized in global LIFO lists.

An abstract address is a forwarding pointer word.

A page contains a page header holding the meta data of the page and the storage space into which objects are allocated. The size of each page is 16KB. All pages are kept aligned in memory. The page header consists of: two pointers used to insert the page into a doubly-linked list, a counter of allocated page-blocks in the page, a reference to the size-class of the page, and a bitmap where each set bit represents a used page-block in the storage space. The bitmap is used for fast location of free and used blocks.

A size-class contains two doubly-linked lists of pages which store the full and the not-full pages, respectively, and a counter of the number of not-full pages.

Global data structures are used to organize data structures which do not belong to a particular size-class. Such are a LIFO list of empty pages and a LIFO list of free abstract addresses. The implementation details that make these data structures concurrent and scalable will be discussed in the following subsections.

Figure 7 presents an overview of all implemented CF versions (leafs of the tree) and introduces terminology.

7.1. Concurrent Non-incremental CF.

We use blocking and non-blocking mechanisms to allow for concurrent use of CF by multiple threads. In particular, locks are used to make the size-class automaton transitions atomic (allocation, deallocation that does not cause compaction, and deallocation with compaction) and non-blocking mechanisms are used to render access to the global LIFO lists atomic and scalable.

In all concurrent implementations size-classes are kept 128B aligned in memory, to avoid cache conflicts of concurrent threads.

We implement locks at two possible levels: size-class locks and page locks. The choice of lock level is evident in the different implementation versions in Figure 7. The page lock level is finer than the size-class lock level, which exists in all implementations. In the presence of page locks, during compaction the size-class lock is released and only the page locks of the source and target page are locked. As a result, other threads may perform memory operations within the size-class that do not affect the source and the target page.

Our managing of the global lists of empty pages and free abstract addresses is inspired by the free list implementation used in [14]. Each of the two lists is organized on two (public and private) levels. Each thread owns one private list (of free elements) which is only accessible to the owner thread. Therefore the access to free elements in the private list needs no synchronization mechanisms. The public list is a list of lists of free elements. Its head contains a version number (used for synchronization between threads) and a reference to the first element (sublist) in the list. Both fields in the list head are updated simultaneously using a double-word compare-and-swap operation, hence the update is atomic. Whenever the reference to the first element changes, the version number increases, which prevents the ABA problem [13]. If a thread needs a free element, then it first accesses its private list. If the private list is empty, then it accesses the public list, in order to fetch the head of the public list of lists. After this, the newly fetched list becomes the private list of the thread. There is also a mechanism that returns elements from a private list to the public list, which is invoked if the private list grows beyond a predefined bound.

There is a slight difference in the implementation of the public list for the list of empty pages and for the list of free abstract addresses. In order to represent the public list in memory, we need for each sublist a pointer to the next sublist. In case of the list of empty pages, we use the empty space of the first page of each sublist to store such a pointer. For the list of free abstract addresses, an additional two-word data structure for storing the pointers is needed.

7.2. **Concurrent Incremental CF.** For incremental compaction, each page-block stores an additional field called compaction-block field. The field has a size of 4B, which is relatively small compared to the size of the page-blocks in size-classes with large page-blocks (larger than 1KB), which are typically subject to incremental compaction. If a page-block becomes a source/target of an incremental compaction operation, then its compaction-block field stores a reference to its corresponding target/source page-block, respectively. Whether a page-block involved in incremental compaction is a source or a target page-block is determined by the status of its page and the status of the page of its compaction block.

In addition, each abstract address contains a flag bit which signals whether the object that the abstract address refers to is a target of a canceled incremental compaction operation. We have discussed deallocation and compaction conflicts in Section 5. In the implementation, a deallocation conflict is detected if the compaction-block field of the page-block under deallocation contains a memory reference. A compaction conflict is also recognized by a memory reference in the compaction-block field of the source page-block under compaction. In case of a deallocation or a compaction conflict, an ongoing compaction operation needs to be canceled. This is done by setting the flag bit in the abstract address of the object that was deallocated and triggered the compaction operation. When the thread in charge of the canceled compaction gets to execute again, it first checks the flag in the abstract address and if the flag is set the thread terminates its compaction operation and releases the abstract address.

7.3. **Local vs. Global Size-classes.** An orthogonal optimization for concurrent CF which improves scalability is using thread-local size-classes. Every thread has a private heap organized in private size-classes. Each thread allocates only in its private heap, but may deallocate shared objects in other thread's heaps. If the percentage of shared objects in the system is low, this optimization leads to less conflicts, thus improving the overall performance.

8. Experiments

We report on micro- and macrobenchmarks with non-concurrent non-incremental CF, concurrent non-incremental CF, and microbenchmarks with concurrent incremental CF.

8.1. **Hardware Setup.** The experiments with *concurrent non-incremental* CF ran on a server machine with two quad-core 2GHz AMD Opteron processors and 16GB of memory. The experiments with non-concurrent and non-incremental CF and concurrent incremental CF were conducted on an XScale PXA 270 CPU with 600MHz and 128MB of memory. The operating system for both machines was Linux with real-time preemption patches applied [33]. On the Opteron machine and the XScale machine the Linux kernel version was 2.6.24 and 2.6.21, respectively. In all experiments the benchmark threads were executed with real-time priorities.

We use two different processors since we are interested in evaluating the behavior of concurrent CF versions both on a multi-core server and on an embedded processor. The multi-core server has high computational power which removes the need of incremental compaction. These experiments are shown in Section 8.2. To demonstrate the behavior of incremental CF in an embedded environment we use the XScale processor. These experiments are shown in Section 8.3. Note that we compare the different concurrent versions of CF among themselves. See [9] for a comparison of the original CF with other allocators.

8.2. **Concurrent Non-incremental CF.** The microbenchmarks all run mutator threads that each allocate 2048 objects of random size, then deallocate the objects, and then start over again. The sizes of allocated objects correspond to the distribution of object sizes allocated in a popular optimizer for programmable logic arrays called Espresso used in several memory allocator performance evaluations, e.g. in [15]. Each microbenchmark runs for ten seconds performing more than one million allocation/deallocation operations.

Figure 8 shows the impact of partial compaction on the allocation throughput of a single thread. Larger partial compaction bounds κ provide higher allocation throughput because of less compaction activity. Independently of κ, the size-class lock configuration performs better then the page-lock configuration since the latter needs locks for both the size-class and the source and target pages.

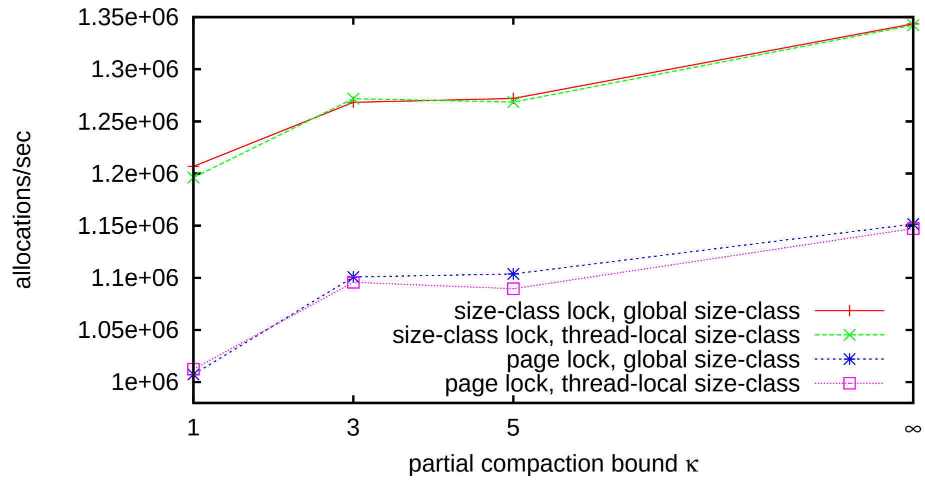
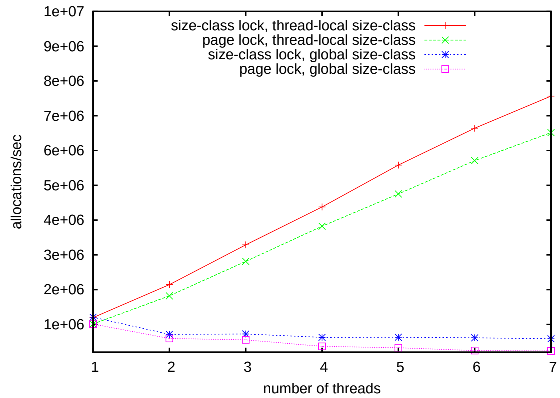


FIGURE 8. Allocation throughput of a single thread with decreasing partial compaction

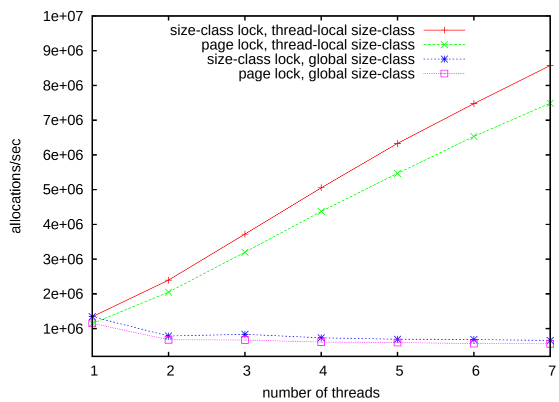
Figure 9 depicts the allocation throughput with an increasing number of threads. Up to seven threads run in parallel on seven cores while the eighth core is used to minimize the influence of collecting data on the performance data. The performance of the fully compacting and the optimized, non-compacting version of CF without abstract addressing (in both cases with no sharing across the thread-local CF instances) are shown in Figures 9(a) and 9(b), respectively. The thread-local size-class versions show linear scalability in the number of threads whereas the global size-class versions neither scale in the fully compacting nor in the non-compacting configurations. Again, the size-class lock configurations result in better allocation throughput than the page lock configurations. Scalability only improves by a constant factor with increasing partial compaction (cf. Figures 9(a) versus 9(b)). Scalability of the thread-local size-class versions depends on the degree of sharing across the thread-local CF instances. Figure 9(c) shows allocation throughput at varying degrees of sharing: mutator threads allocate and deallocate 512 objects periodically according to the Espresso object size distribution. Each mutator frees its own just allocated objects and objects previously allocated by other threads in a ratio that determines the degree of sharing.

The macrobenchmarks are based on Emacs and Hummingbird allocation/deallocation traces [8]. In the Emacs trace about 51% of the allocated objects are of size 40B, 15% are of size 648B, and 11% are of size 104B. The remaining objects of the trace are also of small size. In the Hummingbird trace about 25% of the allocated objects are of size 8B and 23% are of size 32B. The remaining allocation requests vary from 16B to around 38.1MB (object sizes greater than 16KB are ignored here). Hummingbird's allocation behavior is very different from the behavior of a typical mutator where 99% of the objects are of small and similar sizes [15].

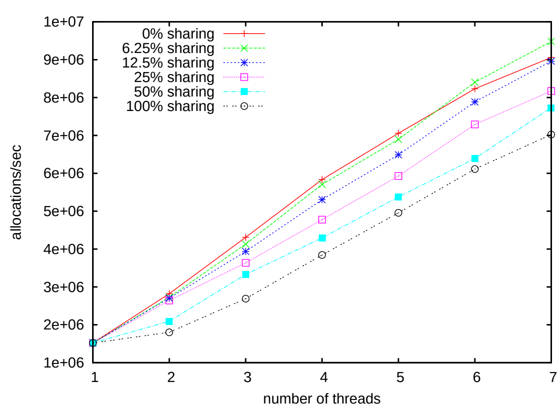
Figure 10 shows the allocation throughput of a single thread running the Hummingbird and Emacs benchmarks. Larger κ values allow the Hummingbird benchmark to allocate more objects per second. In the Emacs benchmark the allocation throughput does not improve for larger κ.



(a) full compaction



(b) optimized, non-compacting



(c) opt., non-comp. with sharing

FIGURE 9. Allocation throughput with an increasing number of threads

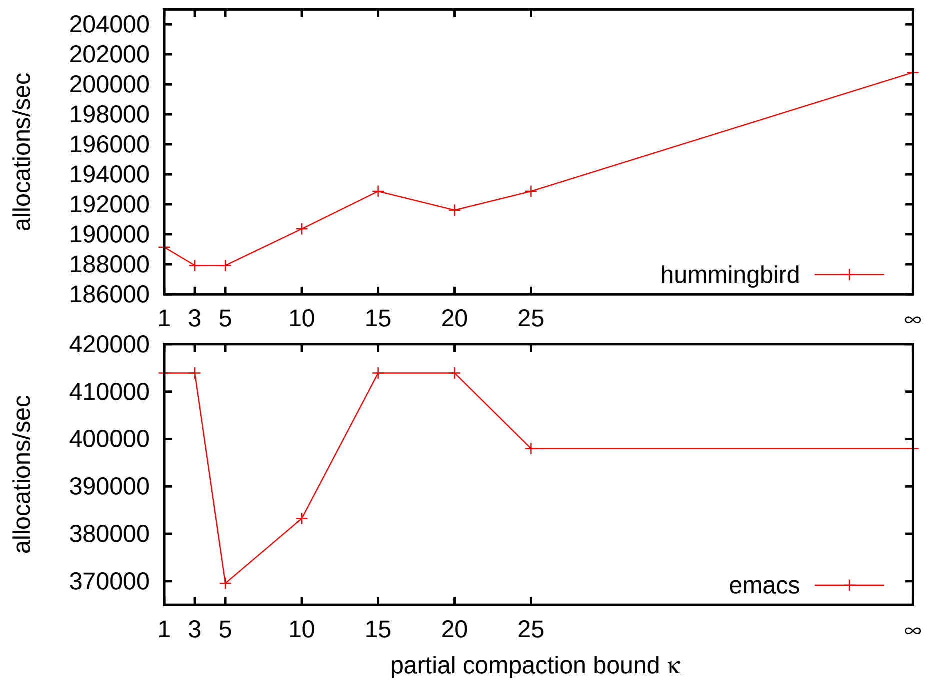


FIGURE 10. Allocation throughput for Hummingbird and Emacs

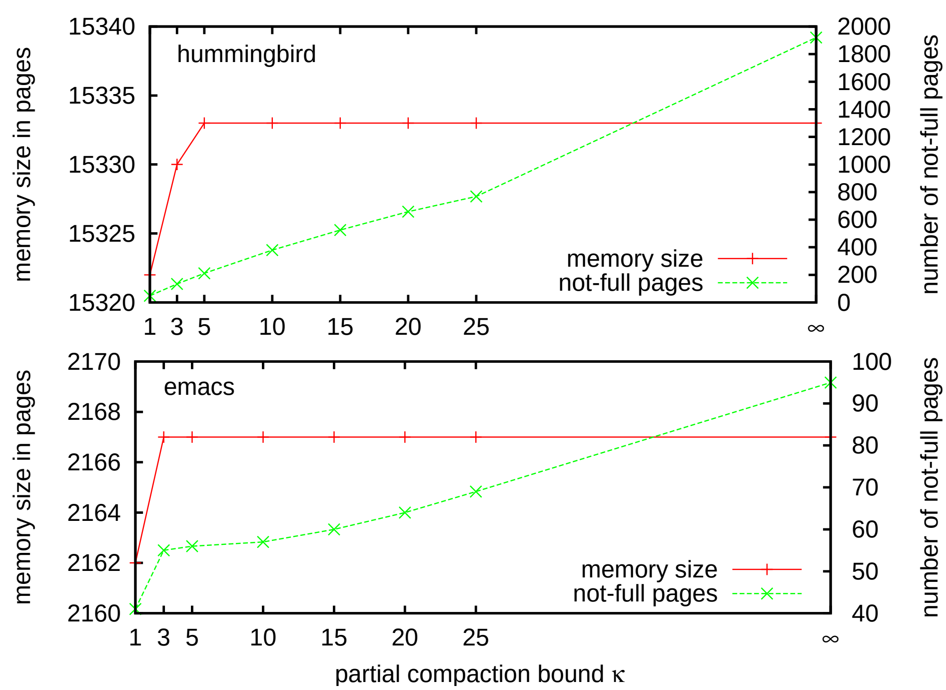


FIGURE 11. Memory usage and size-class fragmentation for Hummingbird and Emacs

	malloc (in clock ticks)				free (in clock ticks)			
	TLSF		CF		TLSF		CF	
	avg time	max time	avg time	max time	avg time	max time	avg time	max time
Emacs	228	93359	260	81662	153	71159	279	74798
Hummingbird	411	109079	529	98820	500	69192	574	79914

Table 2. Performance: TLSF versus optimized, non-compacting CF (without abstract addressing)

Figure 11 shows the required memory size (in number of used pages) and size-class fragmentation (in number of not-full pages) during the execution of the Hummingbird and Emacs traces with increasing κ. As expected, size-class fragmentation increases with increasing κ, whereas the required memory size remains constant for $\kappa \geq 5$ with the Hummingbird trace and $\kappa \geq 3$ with the Emacs trace since most not-full pages with smaller page-block sizes tend to remain relatively full (in line with our probabilistic claims of Section 4).

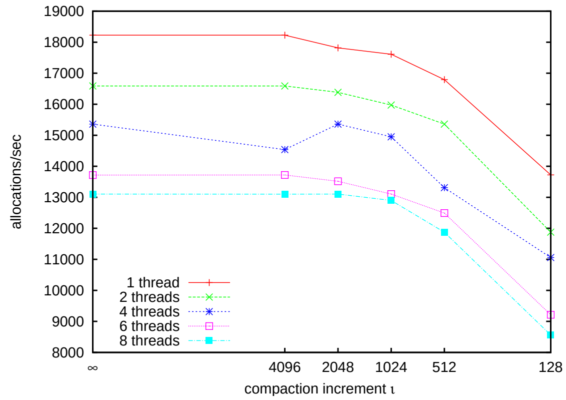
Finally, Table 2 shows the results of macrobenchmarking TLSF [25] and the optimized, non-compacting version of CF without abstract addressing (configured to 16B, and alternatively to 32B, for the smallest page-block size). The temporal performance of malloc and free operations (in clock ticks measured on the Opteron machine) for TLSF and non-compacting CF is similar with TLSF slightly outperforming CF (except for malloc in the worst case where CF is slightly better).

8.3. **Concurrent Incremental CF.** The microbenchmark runs mutator threads allocating and deallocating objects from 16B to 16KB randomly, where 90% of the allocated objects are smaller than 64B [16]. The threads operate on global size classes.

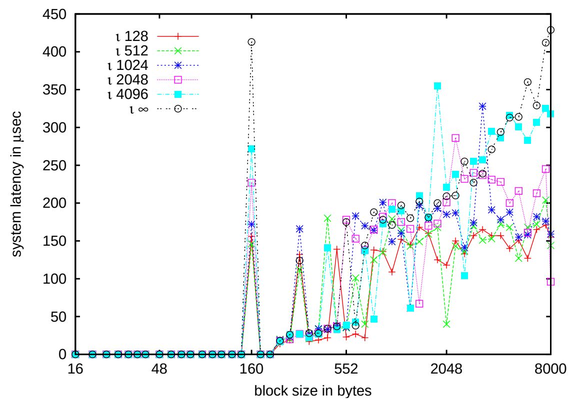
Figure 12(a) shows that the allocation throughput decreases with decreasing compaction increments ι since the incremental compaction overhead increases, due to an increasing number of lock acquire/release operations, administrative data updates, and memory copy interruptions. System latency, shown in Figure 12(b), tends to decrease measurably if page-block sizes larger than around 512B are involved, with decreasing ι. Here, we ran one mutator thread with higher priority than seven other mutator threads, periodically yielding to avoid starvation, and measured the maximum time the higher-priority thread spent in the atomic portion of any incremental compaction operation. True system latency that includes the wait time for locking was too noisy with the version of Linux we used. Transient size-class fragmentation, which is bounded by the number of threads, generally increases slightly with increasing ι as shown in Figure 12(c).

9. Conclusions

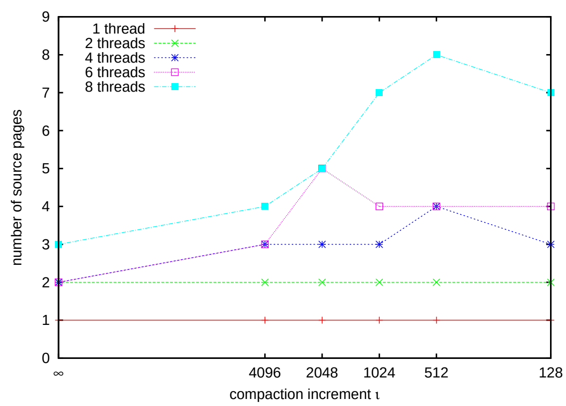
Compact-fit is an explicit, dynamic heap management system that allows, through the notion of partial and incremental compaction, formally relating fragmentation, compaction, throughput, and latency when managing contiguous blocks of memory. We have studied this relationship, formally and experimentally. All versions of CF can be made concurrent and scalable with partial compaction being only a



(a) allocation throughput



(b) system latency with 8 threads



(c) transient size-class fragmentation

FIGURE 12. Allocation throughput, system latency, and transient size-class fragmentation with decreasing compaction increments

constant factor. Scalability rather depends on the degree of sharing and synchronization mechanisms, similar to other heap management systems.

Incremental CF may open up a path to dynamic heap management on memory-constrained systems running high-performance applications that require tight temporal and spatial guarantees, although further studies involving specialized operating system infrastructure for embedded devices may be necessary there.

References

1. *PRISM - probabilistic symbolic model checker*, `http://www.prismmodelchecker.org`.
2. D. F. Bacon, P. Cheng, and V. T. Rajan, *A unified theory of garbage collection*, Proc. OOPSLA, ACM, 2004.
3. David F. Bacon, Perry Cheng, and V. T. Rajan, *Controlling fragmentation and space consumption in the Metronome, a real-time garbage collector for Java*, Proc. LCTES, ACM, 2003.
4. ———, *A real-time garbage collector with low overhead and consistent utilization*, Proc. POPL, ACM, 2003.
5. Ori Ben-Yitzhak, Irit Goft, Elliot K. Kolodner, Kean Kuiper, and Victor Leikehman, *An algorithm for parallel incremental compaction*, Proc. MSP/ISMM, ACM, 2002.
6. A. Bendersky and E. Petrank, *Space overhead bounds for dynamic memory management with partial compaction*, Proc. POPL, ACM, 2011, pp. 475–486.
7. Emery D. Berger, Kathryn S. McKinley, Robert D. Blumofe, and Paul R. Wilson, *Hoard: A scalable memory allocator for multithreaded applications*, Proc. ASPLOS, ACM, 2000.
8. Aniruddha Bohra and Eran Gabber, *Are mallocs free of fragmentation?*, Proc. ATC, USENIX, 2001.
9. Silviu S. Craciunas, Christoph M. Kirsch, Hannes Payer, Ana Sokolova, Horst Stadler, and Robert Staudinger, *A compacting real-time memory management system*, Proc. ATC, USENIX, 2008.
10. Dave Dice and Alex Garthwaite, *Mostly lock-free malloc*, Proc. ISMM, ACM, 2002.
11. Christine H. Flood, David Detlefs, Nir Shavit, and Xiolan Zhang, *Parallel garbage collection for shared memory multiprocessors*, Proc. JVM, USENIX, 2001.
12. L. Gidra, T. Gaël, J. Sopena, and M. Shapiro, *Assessing the scalability of garbage collectors on many cores*, Proc. PLOS, ACM, 2011.
13. M. Herlihy and N. Shavit, *The art of multiprocessor programming*, Morgan Kaufmann Publishers Inc., 2008.
14. Richard L. Hudson, Bratin Saha, Ali-Reza Adl-Tabatabai, and Benjamin C. Hertzberg, *McRT-Malloc: a scalable transactional memory allocator*, Proc. ISMM, ACM, 2006.
15. Mark S. Johnstone and Paul R. Wilson, *The memory fragmentation problem: solved?*, Proc. ISMM, ACM, 1998.
16. M.S. Johnstone and P.R. Wilson, *The memory fragmentation problem: solved?*, Proc. ISMM, ACM, 1998, pp. 26–36.
17. Richard Jones, *The garbage collection page*, `http://www.cs.ukc.ac.uk/people/staff/rej/gc.html`.
18. T. Kalibera, *Replicating real-time garbage collector for java*, Proc. (JTRES), ACM, 2009, pp. 100–109.
19. Tomas Kalibera and Richard Jones, *Handles revisited: optimising performance and memory costs in a real-time collector*, Proc. ISMM, ACM, 2011.
20. Haim Kermany and Erez Petrank, *The Compressor: concurrent, incremental, and parallel compaction*, Proc. PLDI, ACM, 2006.
21. C.M. Kirsch, H. Payer, and H. Röck, *Hierarchical PLABs, CLABs, TLABs in Hotspot*, Proc. International Conference on Systems (ICONS), 2012.
22. Donald E. Knuth, *Fundamental algorithms*, The Art of Computer Programming, Addison-Wesley, 1973.
23. Doug Lea, *A memory allocator*, Unix/Mail/, 6/96, 1996.
24. Miguel Masmano, Ismael Ripoll, and Alfons Crespo, *A comparison of memory allocators for real-time applications*, Proc. JTRES, 2006.

25. Miguel Masmano, Ismael Ripoll, Alfons Crespo, and Jorge Real, *TLSF: A new dynamic memory allocator for real-time systems*, Proc. ECRTS, 2004.
26. Maged M. Michael, *Scalable lock-free dynamic memory allocation*, SIGPLAN **39** (2004), no. 6.
27. K. Nilsen, *Differentiating features of the PERC ^{TM} virtual machine*, `http://www.aonix.com/pdf/PERCWhitePaper.pdf`, 2009.
28. T. Ogasawara, *An algorithm with constant execution time for dynamic storage allocation*, Proc. RTCSA, 1995.
29. Yoav Ossia, Ori Ben-Yitzhak, and Marc Segal, *Mostly concurrent compaction for mark-sweep GC*, Proc. ISMM, 2004.
30. F. Pizlo, L. Ziarek, P. Maj, A.L. Hosking, E. Blanton, and J. Vitek, *Schism: fragmentation-tolerant real-time garbage collection*, Proc. PLDI, ACM, 2010, pp. 146–159.
31. Filip Pizlo, Daniel Frampton, Erez Petrank, and Bjarne Steensgaard, *Stopless: a real-time garbage collector for multiprocessors*, Proc. ISMM, ACM, 2007.
32. Isabelle Puaut, *Real-time performance of dynamic memory allocation algorithms*, Proc. ECRTS, 2002.
33. Real-time Linux, *Real-time Linux*, `http://rt.wiki.kernel.org`.
34. Narendran Sachindran, J. Eliot B. Moss, and Emery D. Berger, *MC^2: high-performance garbage collection for memory-constrained environments*, Proc. OOPSLA, ACM, 2004.
35. M. Schoeberl, *Scheduling of hard real-time garbage collection*, Real-Time Systems **45** (2010), no. 3, 176–213.
36. Fridtjof Siebert, *Real-time garbage collection in the Jamaica VM 3.0*, Proc. JTRES, ACM, 2007.
37. S.-H. Wu, S. A. Smolka, and E. W. Stark, *Composition and behaviors of probabilistic I/O automata*, Theoretical Computer Science **176** (1997).

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