Real-Time Calculus for Scheduling Hard Real-Time Systems

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Motivation

Modular Performance Analysis and Design Space Exploration of Distributed Embedded Systems

System complexity is increasing, a few domains:

- Networks of sensors and actuators
- Building automation
- Car communication
- Environmental monitoring
- ...

Motivation

Fundamental problems:

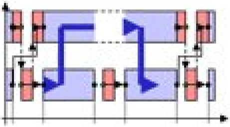
- Handling non-functional and resource constraints
- Design under multiple conflicting criteria
  - Performance vs power consumption
  - Data throughput vs error rate
  - ...
- Trade-off between average performance and predictability

Motivation

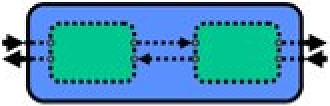
Modular Design Strategies

Predictable Wireless Embedded Systems

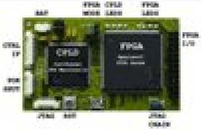
Giotto



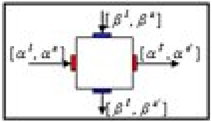
Interface Theories



HW/SW Task Machine



Real-Time Calculus



Motivation

Modular Design Strategies – Vision

Finite resources

- Buffer space
- Energy
- Communication
- Processing power
- Time
- ...

Requirements:

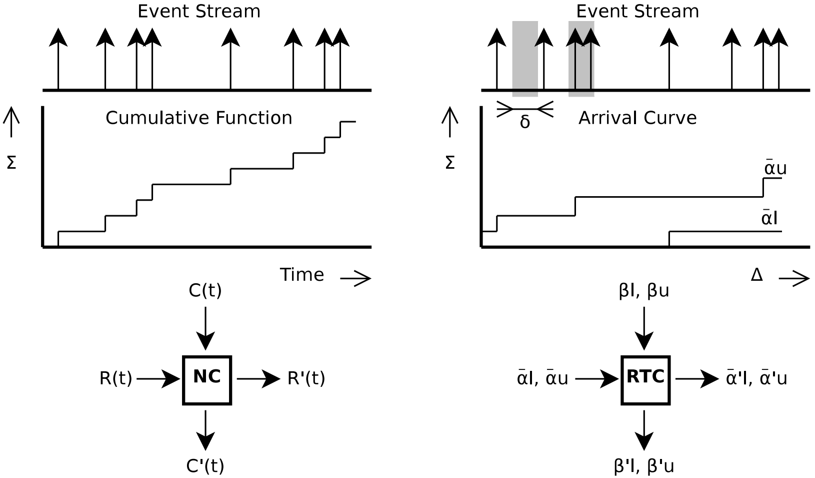
- Distributed systems
- Need for run-time adaptability
- Allow for guarantees

Real-Time Calculus

In a Nutshell

- Deterministic queueing theory
- Hard upper and lower bounds are *always* found
- No insight into average load of a system
- Extension of Network Calculus (R. L. Cruz, 1991)

Real-Time Calculus



Network Calculus **Real-Time Calculus**

Real-Time Calculus

Relationship

Network Calculus

- Time (absolute)
- Cumulative Function
- Event stream $R(t)$
- Resources $C(t)$

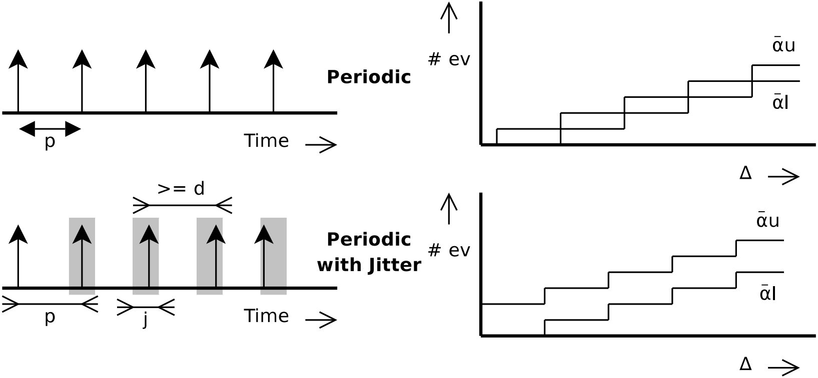
Real-Time Calculus

- Interval size Δ (relative time)
- Arrival Curve
- Event curve α_l, α_u
- Resource curve β_l, β_u

Real-Time Calculus

- Nodes are modeled as pure mathematical functions
- Computation resources: CPU cycles
- Communication resources: Transported number of bits

Real-Time Calculus



Real-Time Calculus

Proposition 1: Basic Model For a processing node characterised by the capacity function $C(t)$ and the incoming requests function $R(t)$ we have $C'(t) = C(t) - R(t)$ and

$$R'(t) = \min_{0 \leq u \leq t} \{R(u) + C(t) - C(u)\}$$

Real-Time Calculus

Bounds

Proposition 2: Request Curve For a given request function R, the minimum request curve α_r can be calculated as

$$\alpha_r = \max_{u \geq 0}\{R(\Delta + u) - R(u)\}$$

Proposition 3: Delivery Curve For a given capacity function C, the maximum delivery curve β can be calculated as

$$\beta = \min_{u \geq 0}\{C(\Delta + u) - C(u)\}$$

Real-Time Calculus

Processing of Bounds

Proposition 4: Remaining Delivery Curve Given request and capacity functions R and C, bounded by the request and delivery curves α_r and β respectively, C' according to Prop. 1 is then bounded by the delivery curve

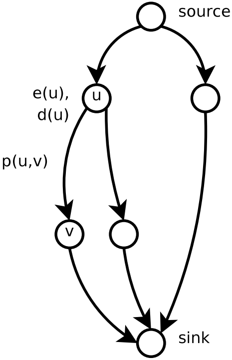
$$\beta'(\Delta) = \max_{0 \leq u \leq \Delta} \{\beta(u) - \alpha_r(u)\}$$

Proposition 5: Delivered Computation Bounds Given request and capacity functions R and C, bounded by the request and delivery curves α_r and β respectively, R' according to Prop. 1 is then bounded by the request curve

$$\alpha'(\Delta) = \max_{u \geq 0} \{\alpha_r(\Delta + u) - \beta(u)\}$$

Real-Time Calculus

Calculation of Bounds



Task Graph with Conditional Branches

c.f. S. Baruah, 1998

- Task graph: DAG with unique source and sync vertex
- Vertex u is subtask
- Associated with pair $(e(u), d(u))$
- Requires $e(u)$ time, finishes $d(u)$ after being triggered
- Directed edge (u, v) is control flow
- $p(u, v)$ is min time before v can be triggered

Real-Time Calculus

Calculation of Bounds

$L^{j+1} = (e(j+1), d(j+1))$
for each edge $(k, j+1)$ **do**
  for each tuple $(f, \Delta) \in L^k$ **do**
    $L^{j+1} \cup$
      $\{(f + e(j+1), \Delta + p(k, j+1)$
        $+ d(j+1) - d(k))\}$
$L(j+1) = L^{j+1} \cup L(j)$
Reduce sets $L(j+1)$ and L^{j+1}

- Incremental algorithm
- L^i set of tuples (f, Δ)
  sequences of executions
  i is last subtask
- $L(i)$ list of tuples
  subtasks up to, including i

Reduce removed tuples with less restrictive bounds

Real-Time Calculus

Theorem 1 Given a task graph of n vertices, $\alpha_d(\Delta)$ can be computed in $O(n^3)$ time if the execution times of the subtasks are equal, and in general it can be computed in pseudo-polynomial time.

Real-Time Calculus

Proposition 6: Schedulability Test The capacity offered by a task T_i can be described by the remaining delivery curve β' according to Prop. 4 where $\alpha_r = \sum_{j=1}^{i-1}$. Task Ti meets all its deadlines if

$$(\forall \Delta)(\beta'(\Delta) \geq \alpha_d^i(\Delta))$$

Conclusion

- Terse but concise paper
- Key contribution is algorithm in polynomial time

- Critique: request/arrival model and task graph model could be integrated better