Compositional Real-Time Scheduling Framework

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Outline

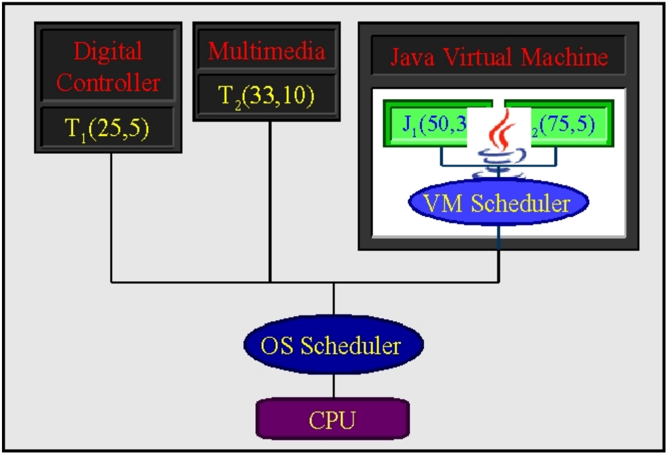
Problem Statement

- develop a compositional real-time scheduling framework
- The two essential problems in developing such a framework are :
  - to abstract the collective real-time requirements of a component as a single real-time requirement - *scheduling interface*
  - to compose the component demand abstraction results into the system-level real-time requirement - *scheduling component composition*.

Ideally...

...the single real-time requirement is satisfied if and only if the set of components are satisfied.

Compositional Scheduling Framework



Overview
Scheduling

- Scheduling assigns resources to workloads by scheduling algorithms
- Scheduling Component Model :

$$C(W, R, A)$$

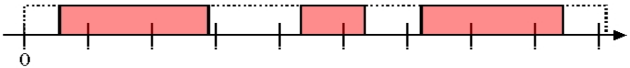
  - W : workload model
  - R : resource model
  - A : scheduling algorithm

Overview

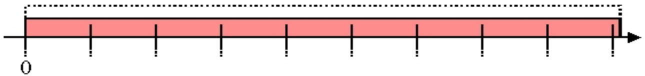
Resource

- Dedicated resource : always available at full capacity

- Shared resource : not a dedicated resource



- Non-time-sharing : available at fractional capacity



Prerequisites

- *Resource demand* of $C(W, R, A)$ represents the collective resource requirements that W requests under A.
- *Demand bound function* $dbf_A(W, t, i)$ is the maximum possible resource demand that W requests to satisfy the timing requirements of task i under A within t.
- *Resource supply* of resource model R is the amount of resource allocations that R provides.
- *Supply bound function* $sbf_R(t)$ is the minimum possible resource supplies that R provides during t.

A resource model R is said to satisfy a resource demand of W under A if

$$dbf_A(W, t, i) \leq sbf_R(t)$$

Prerequisites

Schedulability

A scheduling component $C(W, R, A)$ is said to be schedulable if and only if

$$\forall i \in W, \forall t \Longrightarrow dbf_A(W, t, i) \leq sbf_R(t)$$

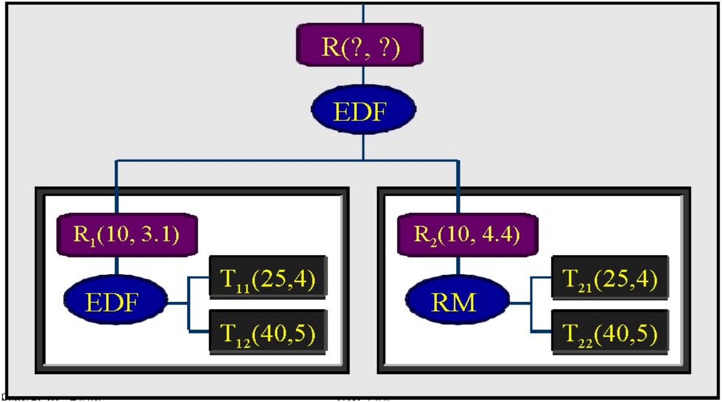
Problem statement

Given W and A such that $C(W, R_p, A)$ is schedulable, where R_p is a dedicated resource, the problem is to find an optimal shared resource model R such that $C(W, R, A)$ is schedulable. R is the scheduling interface of C.

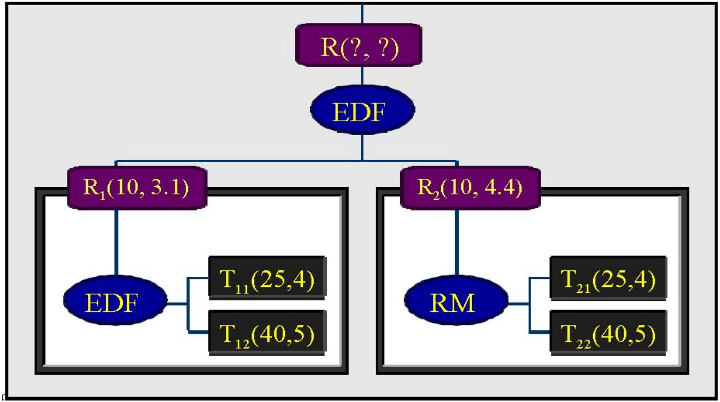
Formal problem statement

Given two scheduling components $C(W_1, R_1, A_1)$ and $C(W_2, R_2, A_2)$ such that $C(W, R_p, A)$ is schedulable, where $W = \{R_1, R_2\}$ and R_p is a dedicated resource, the problem is to find a optimal R such that $C(W, R, A)$ is schedulable.

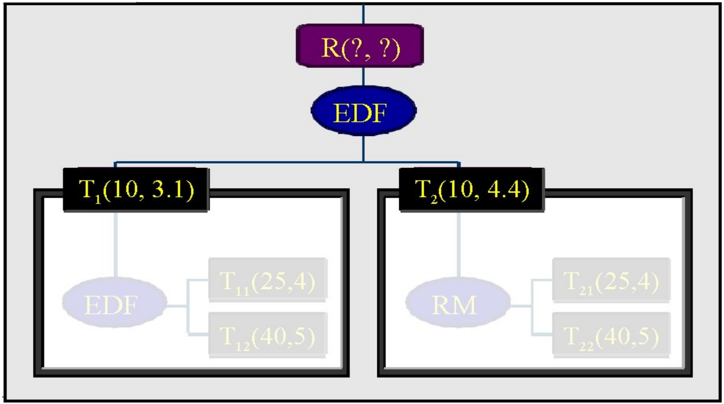
Example



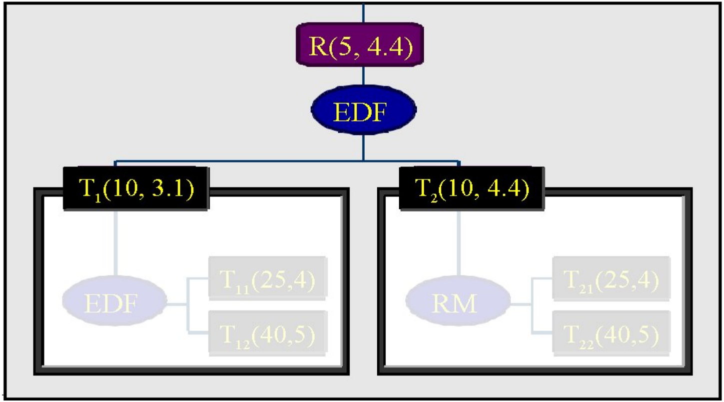
Example



Example



Example



Models

- How do we define a scheduling interface model?
  - In previous work the periodic resource model is defined(that Harald presented)

    $$\Gamma(\Pi, \Theta)$$

    specifies a periodic behavior of time-shared resource allocation and utilization bounds under EDF and RM.
  - bounded-delay model(presented further in this paper)

    $$\Phi(\alpha, \Delta)$$

- Using the 2 models as scheduling interface models the goal is to abstract a set of tasks into a single periodic or bounded-delay task.

Compositional Framework Models

Assumptions:

- periodic task model $T(p, e)$, p is a period and e is an execution time requirement ($e \leq p$).
- task utilization U_T is $\frac{e}{p}$.
- for a workload set $W = \{T_i\}$, a workload utilization U_W is $\sum_{T_i \in W} U_{T_i}$.
- let P_{min} be the smallest period in W, i.e. $P_{min} = min_{T_i \in W}\{p_i\}$.
- each task in independent and preemptive.
- as A we consider EDF and RM.
- as R we consider a time-shared resource model.

The Model

Bounded delay resource model : Maximum delay Δ that a partition must wait to get its share α of the resource for any time interval starting at any point in time

$$\Phi(\alpha, \Delta)$$

where α is an available factor(resource capacity) $0 \leq \alpha \leq 1$ and Δ is a partition delay bound $0 \leq \Delta$.
$\Phi(\alpha, \Delta)$ is defined to characterize the property:
$\forall t_1, \forall t_2 \geq t_1, \forall d \leq \Delta$
$(t_2 - t_1 - d)\alpha \leq supply_\Phi(t_1, t_2) \leq (t_2 - t_1 + d)\alpha$
$sbf_\Phi(t) = \alpha(t - \Delta), t \geq \Delta$ and 0 otherwise

Example

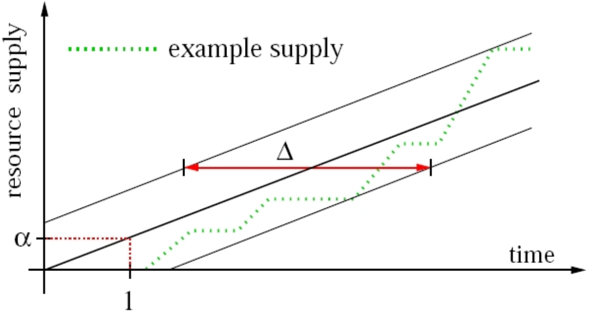


Figure 1. Bounded-delay model: example.

Periodic Workload Model

- Generalize the schedulability conditions for use in any partitioned resource model.
- The resource model must calculate its supply bound function accurately.
- $dbf_{EDF}(W, t) = \Sigma_{T_i \in W}(\lfloor \frac{t-D_i}{p_i} \rfloor + 1) \cdot e_i$ (*Baruah et al. [2]*)
- $dbf_{RM}(W, t, i) = e_i + \Sigma_{T_i \in HP_W(i)} \lceil \frac{t}{p_k} \rceil \cdot e_k$ (*Lehoczky et al. [8]*)

Periodic Workload Model

- $C(W, R, A)$ is schedulable under EDF if and only if

$$\forall 0 < t \leq 2 \cdot LCM_W + D_{max}, dbf_{EDF}(w, t) \leq sbf_R(t)$$

  where LCM_W is the least common multiple of p_i for all $T_i \in W$ and D_{max} is the maximum relative deadline D_i for all $T_i \in W$

- $C(W, R, A)$ is schedulable under RM if and only if

$$\forall T_i \in W, \exists 0 < t \leq p_i, dbf_{RM}(W, t, i) \leq sbf_R(t)$$

Bounded Delay Workload Model

- Transform each bounded-delay workload model into a periodic workload model and analyze schedulability under EDF and RM.
- Minimum acceptable resource demand for an interval t is $dbf(\Phi, t) = \alpha \cdot (t - \Delta) \leq demand_{\Phi}(t)$
- For a periodic workload model $T(p, e)$ we have $dbf(T, t)$
- To transform $\Phi(\alpha, \Delta)$ in $T(p, e)$ we must ensure that $dbf(\Phi, t) \leq dbf(T, t)$ for all t.

Bounded Delay Workload Model

Feasibility and schedulability

- We take an extended bounded-delay workload model $\overline{\Phi}(\alpha, \Delta, Q)$ (*see Feng and Mok [5]*)

Theorem

A component $C(W, R, A)$ is feasible, where $W = \{\overline{\Phi_i}(\alpha_i, \Delta_i, Q)\}, 1 \leq i \leq n$ and $R = \overline{\Phi}(\alpha, \Delta, Q)$ if and only if

$$\forall t > 0, \Sigma_{i=1}^{n} dbf(\overline{\Phi_i}, t) \leq sbf_{\overline{\Phi}}(t)$$

Proof in the paper.

- Schedulable utilization bound of partitioned resource models
- Computing the utilization bound takes a constant amount of time, less than computing *dbf*
- The paper introduces utilization bounds for bounded-delay resource model

Theorem

A component $C(W, R, A)$ is schedulable, where $W = \{T_i(p_i, e_i)\}, R = \Phi(\alpha, \Delta)$, under EDF if

$$U_W \leq \alpha(1 - \frac{\Delta}{P_{min}}), P_{min} = min_{T_i \in W}\{p_i\}$$

Theorem

A component $C(W, R, A)$ is schedulable, where $W = \{T_i(p_i, e_i)\}, R = \Phi(\alpha, \Delta)$, under RM if

$$U_W \leq \alpha(n(\sqrt[n]{2} - 1) - \frac{\Delta}{2^{(n-1)/n} \cdot P_{min}}), P_{min} = min_{T_i \in W}\{p_i\}$$

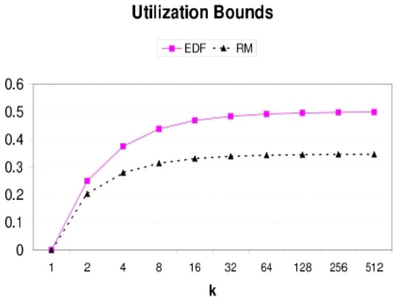
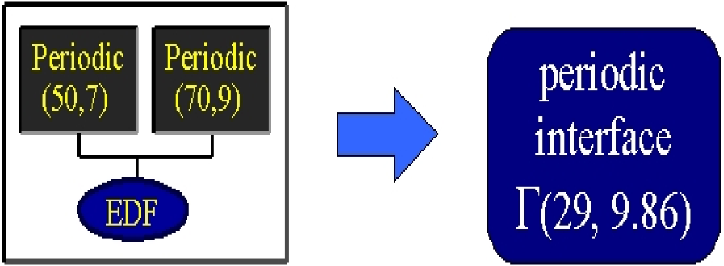
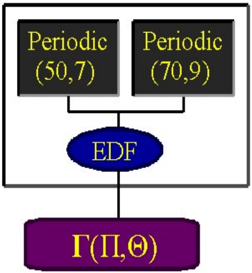


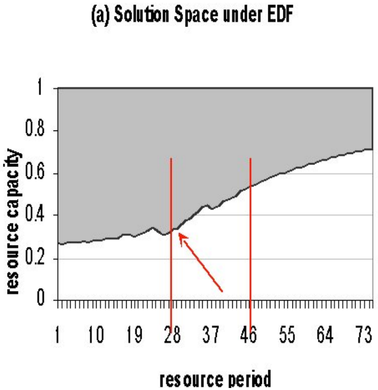
Figure 5. Utilization bounds of a bounded-delay resource model $\Phi(\alpha, \Delta)$, where $\alpha = 0.5$, as a function of k, where $k = P_{min}/\Delta$, under EDF and RM scheduling

As *k* increases the utilization bounds converge to their limits α under EDF and *log*2 $\cdot\, \alpha$ under RM.

Given a workload set W and a scheduling algorithm A such that the scheduling component $C(W, R_p, A)$ is schedulable the problem is to find an optimal resource model R such that $C(W, R, A)$ is schedulable.
We define the optimality criteria as minimizing the resource capacity requirement of a solution when a resource period bound is given.




Periodic (50,7)
Periodic (70,9)
EDF
$\Gamma(\Pi,\Theta)$

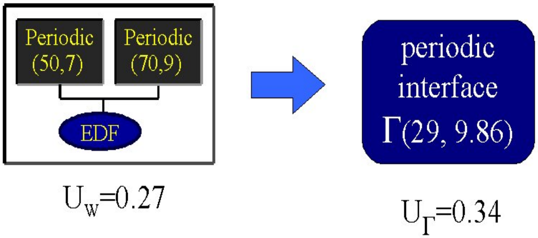

(a) Solution Space under EDF
resource capacity
1
0.8
0.6
0.4
0.2
0
1
10
19
28
37
46
55
64
73
resource period

Simulation

- Workload size(number of tasks in W) : $2, 4, 8, 16, 32, 64, 128$
- Workload utilization : $0.1, 0.2, \ldots, 0.7$
- Task model : Each task has a period p randomly generated in the range $[5, 100]$ and an execution time e in $[1, 40]$
- Scheduling algorithm : EDF or RM
- Delay Bound (Δ) : is determined such that $k = 2, 4, 8, 16, 32, 64$ where $k = \frac{P_{min}}{\Delta}$ and P_{min} is the smallest task period.

Simulation

For a scheduling component $C(W, R, A)$, its abstraction overhead (O_Γ) is $\frac{U_\Gamma}{U_W} - 1$



Simulation

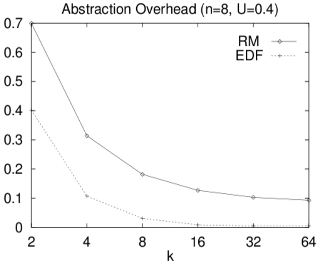


Figure 6. Scheduling component abstraction overheads as a function of k under EDF and RM scheduling, where $k = P_{min}/\Delta$.

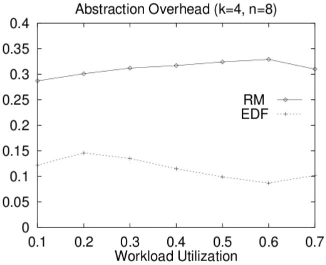


Figure 7. Scheduling component abstraction overheads as a function of workload utilization under EDF and RM scheduling.

Simulation

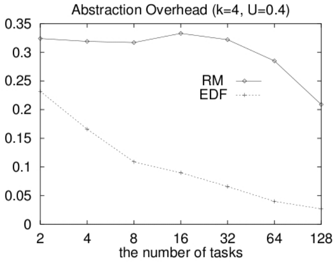


Figure 8. Scheduling component abstraction overheads as a function of workload size under EDF and RM scheduling.

Conclusion

- a bounded-delay model can be used as scheduling interface model for compositional scheduling frameworks
- defined and provided solutions to the problem of developing a compositional real-time scheduling framework
- Drawback : limitation to the tasks, we assume that they are independent

Thank you

Thank you!
Any questions?

Supply bound function

- $sbf_{\Phi^{\sim}}(t) = t - t_k^* + (k-1) \cdot Q$ if $t \in [t_k^*, t_k^* + Q]$, and $k \cdot Q$ if $t \in [t_k^* + Q, t_{k+1}^*]$ where Q is the minimum scheduling quantum, $t_k^* = t_k - \lfloor \frac{t_k}{Q} \rfloor Q$ such that $t_k = (k-1)\frac{Q}{\alpha} + \Delta, k = 1, 2, \ldots$

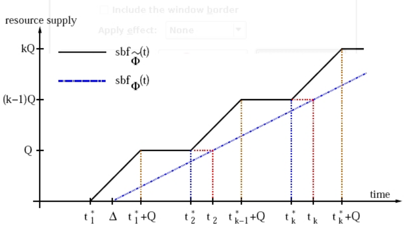


Figure 2. Extended bounded-delay model with scheduling quantum: supply bound function.