Oplossingen Tentamen Procesalgebra 28 november 2002

1. i.
$(a.\epsilon + a.b.\epsilon)\|c.\epsilon =$
$(a.\epsilon + a.b.\epsilon) \mathbin{\|\!\_} c.\epsilon + c.\epsilon \mathbin{\|\!\_} (a.\epsilon + a.b.\epsilon) + (a.\epsilon + a.b.\epsilon)|c.\epsilon =$
$a.\epsilon \mathbin{\|\!\_} c.\epsilon + a.b.\epsilon \mathbin{\|\!\_} c.\epsilon + c.(a.\epsilon + a.b.\epsilon) + a.\epsilon|c.\epsilon + a.b.\epsilon|c.\epsilon =$
$a.c.\epsilon + a.(b.\epsilon\|c.\epsilon) + c.(a.\epsilon + a.b.\epsilon) + \delta + \delta =$
$a.c.\epsilon + a.(b.c.\epsilon + c.b.\epsilon) + c.(a.\epsilon + a.b.\epsilon)$

1. ii.
$\partial_{\{a,b\}}(a.\epsilon\|b.\epsilon\|b.\epsilon) =$
$\partial_{\{a,b\}}(a.\epsilon \mathbin{\|\!\_} (b.\epsilon\|b.\epsilon) + b.\epsilon \mathbin{\|\!\_} (a.\epsilon\|b.\epsilon) + b.\epsilon \mathbin{\|\!\_} (a.\epsilon\|b.\epsilon) + (a.\epsilon|b.\epsilon) \mathbin{\|\!\_} b.\epsilon +$
$(a.\epsilon|b.\epsilon) \mathbin{\|\!\_} b.\epsilon + (b.\epsilon|b.\epsilon) \mathbin{\|\!\_} a.\epsilon) =$
$\partial_{\{a,b\}}(a.b.b.\epsilon + b.(a.b.\epsilon + b.a.\epsilon) + c.b.\epsilon) =$
$\delta + \delta + c.\delta =$
$c.\delta$

1. iii.
$\tau_a(a.b.\epsilon\|a.b.\epsilon) =$
$\tau_a(a.b.\epsilon \mathbin{\|\!\_} a.b.\epsilon + a.b.\epsilon \mathbin{\|\!\_} a.b.\epsilon + a.b.\epsilon|a.b.\epsilon) =$
$\tau_a(a.(b.\epsilon\|a.b.\epsilon) + \delta) =$
$\tau_a(a.(b.\epsilon \mathbin{\|\!\_} a.b.\epsilon + a.b.\epsilon \mathbin{\|\!\_} b.\epsilon + b.\epsilon|a.b.\epsilon)) =$
$\tau_a(a.(b.a.b.\epsilon + a.(b.\epsilon\|b.\epsilon) + c.(\epsilon\|b.\epsilon))) =$
$\tau_a(a.(b.a.b.\epsilon + a.b.b.\epsilon + c.b.\epsilon)) =$
$\tau.(b.\tau.b.\epsilon + \tau.b.b.\epsilon + c.b.\epsilon) =$
$\tau.(b.b.\epsilon + \tau.b.b.\epsilon + c.b.\epsilon)$

1. iv.
$\pi_2(a.b.\delta\|\tau.\delta) =$
$\pi_2(a.b.\delta \mathbin{\|\!\_} \tau.\delta + \tau.\delta \mathbin{\|\!\_} a.b.\delta + a.b.\delta|\tau.\delta) =$
$\pi_2(a.(b.\delta\|\tau.\delta) + \tau.a.b.\delta + \delta) =$
$a.\pi_1(b.\delta\|\tau.\delta) + \tau.\pi_2(a.b.\delta) =$
$a.\pi_1(b.\tau.\delta + \tau.b.\delta) + \tau.a.b.\delta =$
$a.(b.\pi_0(\tau.\delta) + \tau.\pi_1(b.\delta)) + \tau.a.b.\delta =$
$a.(b.\tau.\delta + \tau.b.\delta) + \tau.a.b.\delta =$
$a.(\tau.(b.\delta + \delta) + b.\delta) + \tau.a.b.\delta =$
$a.b.\delta + \tau.a.b.\delta$

2.i.

[illegible]

2.ii

[illegible]

2.iii.

3.i.
Omdat er voor alle termen x, y basistermen x', y' zijn met $x = x', y = y'$, is het voldoende om de eigenschap aan te tonen voor basistermen. We bewijzen de eigenschap met inductie naar de structuur van x.

Basis
$x \equiv \epsilon$. Dan $\epsilon|\tau.y = \delta$.
$x \equiv \delta$. Dan $\delta|\tau.y = \delta$.

IH $x'|\tau.y = \delta$, $x''|\tau.y = \delta$ voor alle y.

$x = a.x'$: $\quad a.x'|\tau.y = \delta$

$x = x' + x''$: $(x' + x'')|\tau.y = x'|\tau.y + x''|\tau.y = \delta + \delta = \delta$. $\square$

3.ii.
Tegenvoorbeeld: $x = \epsilon, y = \epsilon$. Dan $x \lfloor\!\lfloor \tau.y = \epsilon \lfloor\!\lfloor \tau.\epsilon = \delta$, maar $x \lfloor\!\lfloor y = \epsilon \lfloor\!\lfloor \epsilon = \epsilon$.

3.iii.
$\tau.(\tau.x\|y) =$
$\tau.\tau.x \lfloor\!\lfloor y =$
$\tau.(\tau.(x + \delta) + \delta) \lfloor\!\lfloor y =$
$\tau.(x + \delta) \lfloor\!\lfloor y =$
$\tau.(x\|y)$.

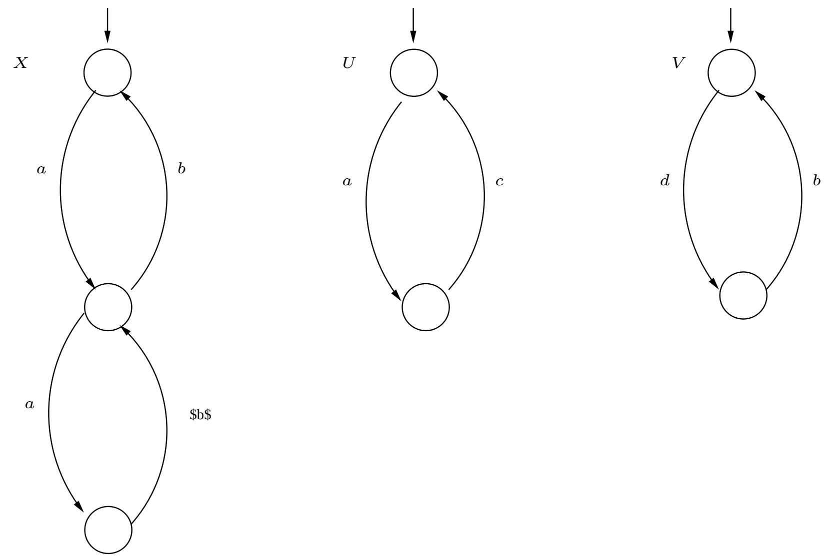
3.iv.
$\tau.x\|\tau.y =$
$\tau.x \lfloor\!\lfloor \tau.y + \tau.y \lfloor\!\lfloor \tau.x + \tau.x|\tau.y =$
$\tau.(x\|\tau.y) + \tau.(y\|\tau.x) + \delta =$
$\tau.(\tau.y\|x) + \tau.(\tau.x\|y) =$

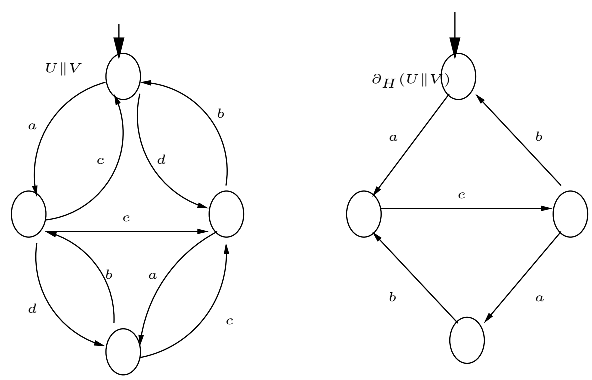
$\tau.(y \| x) + \tau.(x \| y) =$
$\tau.(x \| y)$

4.i.
$U \| V = a.(U' \| V) + d.(U \| V')$
$U' \| V = c.(U \| V) + d.(U' \| V') + e.(U \| V')$
$U \| V' = a.(U' \| V') + b.(U \| V)$
$U' \| V' = c.(U \| V') + b.(U' \| V)$

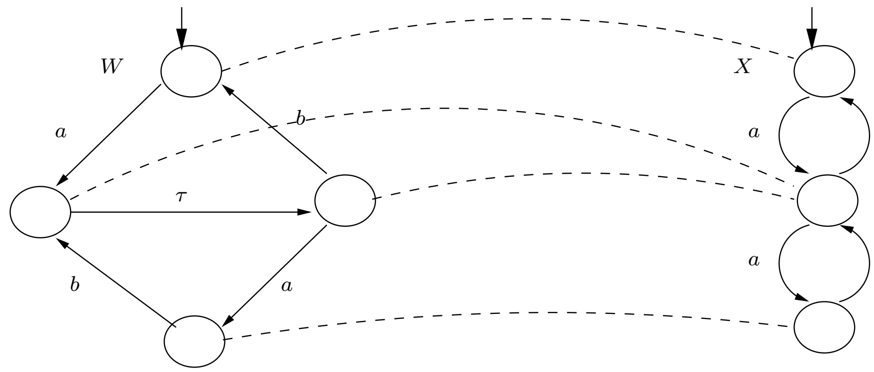
$\partial_H(U \| V) = a.\partial_H(U' \| V)$
$\partial_H(U' \| V) = e.\partial_H(U \| V')$
$\partial_H(U \| V') = a.\partial_H(U' \| V') + b.\partial_H(U \| V)$
$\partial_H(U' \| V') = b.\partial_H(U \| V')$

Dus de procesgrafen voor $X, U, V, U \| V, \partial_H(U \| V)$ zijn:





4.ii.
De procesgrafen voor X en W zijn rb-bisimilair.



4.iii.
Van 4.i. hebben we dat
$W = a.W'$
$W' = \tau.W''$
$W'' = a.W''' + b.W$
$W''' = b.W'$

of

$W = a.\tau.W'' = a.W''$
$W'' = a.W''' + b.W$
$W''' = b.W'$
en het is nu duidelijk dat $X = W$.

4.iv.
We bewijzen dat $\pi_n(X) = X_n, \pi_n(X') = X'_n$ en $\pi_n(X'') = X''_n$ voor alle $n \geq 1$ met inductie naar n.

Basis $\pi_1(X) = a.\delta = X_1, \pi_1(X') = a.\delta + b.\delta = X'_1, \pi_1(X'') = b.\delta = X''_1$

IH $\pi_n(X) = X_n, \pi_n(X') = X'_n, \pi_n(X'') = X''_n$.

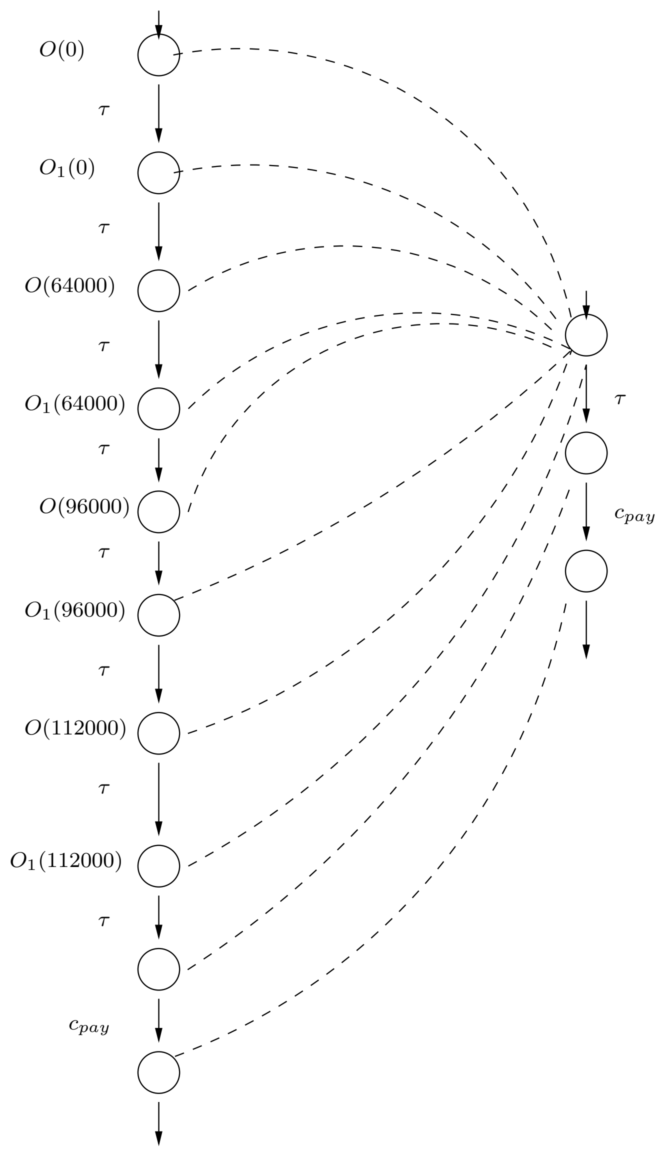
Dan $\pi_{n+1}(X) = a.\pi_n(X'') + b.\pi_n(X) = a.X''_n + b.X_n = X_{n+1}$.
$\pi_{n+1}(X') = a.\pi_n(X'') + b.\pi_n(X) = a.X''_n + b.X_n = X'_{n+1}$.
$\pi_{n+1}(X'') = b.\pi_n(X') = b.X'_n = X''_{n+1}$. □

5.i.
$X(b) = \partial_H(P_1(b) \| P_2) = c_{offer}.\partial_H(Q_1(b) \| Q_2(b)) = c_{offer}.X_1(b)$
$X_1(b) = c_{reject}.\partial_H(P_1((b + T + 1)div2) \| P_2) = X_1(b) =$

$$= \begin{cases} c_{reject}.X((b + T + 1)div2), & \text{als } 0 \leq b < B \\ c_{accept}.c_{pay}.\epsilon, & \text{als } b \geq B \end{cases}$$

5.ii.

$O(b) = \tau.\tau_I(\partial_H(Q_1(b) \| Q_2(b))) = \tau.O_1(b)$
$O_1(b) = \tau.\tau_I(\partial_H(P_1((b+T+1)div2) \| P_2)) = \tau.O((b+T+1)div2),\ 0 \leq b < B$
$O_1(b) = \tau.c_{pay}.\epsilon,\ b \geq B$
Voor $T = 128.000, B = 112.000$ is de procesgraaf van $O(0)$ de volgende:



Het is rb-bisimilair met de graaf voor $\tau.c_{pay}.\epsilon$.

5.iii. Als $T < B$ vindt er geen verkoop plaats.