

Formale Systeme Proseminar

Tasks for Week 8, 26.11.2020

Task 1 Is the following statement true? If yes, prove it with a calculation; if not, give a counter example.

$$\neg \exists_x [P : Q] \stackrel{val}{=} \forall_x [Q : P]$$

Task 2 Prove with a calculation that the following formula is a tautology.

$$\forall_x [P : Q \Rightarrow R] \Rightarrow (\forall_x [P : Q] \Rightarrow \forall_x [P : R])$$

Task 3 Show with derivations that the following formula is a tautology

$$((P \Rightarrow Q) \Rightarrow P) \Rightarrow ((P \Rightarrow Q) \Rightarrow Q)$$

Task 4 For each of the line numbers of your solution to Task 3, say where the proposition which occurs on that line is valid (i.e. allowed to be used).

Task 5 Give logical derivation of the following tautology.

$$(P \wedge (Q \Rightarrow R)) \Rightarrow ((P \Rightarrow Q) \Rightarrow (P \wedge R))$$

Task 6 Show with derivations that the following formula is a tautology

$$\neg(P \Rightarrow Q) \Rightarrow \neg Q$$

Task 7 Give logical derivation of the following tautology

$$(P \wedge \neg Q) \Rightarrow \neg(P \Rightarrow Q)$$

Task 8 Give logical derivation of the following tautology

$$(P \Rightarrow Q) \vee P$$