

# Formale Systeme Proseminar

Tasks for Week 4, 29.10.2020

**Task 1** Check in each of the following cases whether the given rule is correct. If it is, give arguments to show this. If it is not, give a counter example.

- (a) There are K's which are also M's  
All K's are L's

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There are L's which are M's

- (b) No one K is an M  
All K's are L's

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No one L is an M

**Task 2** For each of the following concrete propositions, write an abstract proposition which corresponds to it:

- (a) I love you and will always be true to you.  
(b) If it is raining, then I will stay home and watch a movie.  
(c)  $x^2 > 4$  if, and only if,  $x > 2$  or  $x < -2$ .  
(d) I will go climbing if you bring a rope.

**Task 3** Give the following propositions in words again, with 'it is raining' for  $a$ , 'it is windy' for  $b$ , and 'I am wet' for  $c$ .

- (a)  $a \wedge \neg b$   
(b)  $\neg(a \vee b)$   
(c)  $(a \Rightarrow c) \vee (b \Rightarrow \neg a)$ .  
(d)  $\neg\neg a$ .

**Task 4** Draw the trees of the following abstract propositions and give the main symbol for each of them.

- (a)  $(a \Rightarrow (b \Rightarrow a))$   
(b)  $((\neg(a \Rightarrow b)) \Leftrightarrow (a \wedge (\neg b)))$   
(c)  $((\neg(\neg a)) \Rightarrow ((\neg a) \wedge b))$

(d)  $(a \Rightarrow ((b \wedge a) \vee c))$ .

**Task 5** Drop as many parentheses as possible from the abstract propositions of Task 4.

**Task 6** Give the truth tables of the abstract propositions of Task 4.

**Task 7** For which values of  $a, b$ , and  $c$  one gets 0 in the truth-table of

$$(a \wedge (b \Rightarrow c)) \Rightarrow ((b \Rightarrow a) \wedge c) ?$$

**Task 8** Check whether the following two propositions are equivalent:

- (a)  $\neg(b \vee \neg c)$  and  $\neg b \wedge c$
- (b)  $a \Rightarrow b$  and  $\neg a \Rightarrow \neg b$
- (c)  $(a \vee b) \wedge a$  and  $a$
- (d)  $(a \vee b) \wedge b$  and  $(b \wedge c) \vee (b \wedge \neg c)$ .

**Task 9** Give an example of a tautology (i.e., an abstract proposition that is always true independent of the truth-values of its variables) with only one proposition variable  $a$  and with only parentheses and

- (a) connective  $\Rightarrow$
- (b) connectives  $\vee$  and  $\neg$
- (c) connectives  $\wedge$  and  $\neg$
- (d) connective  $\Leftrightarrow$ .