

# Formale Systeme Proseminar

Tasks for Week 15, 28.1.2021

**Task 1** Construct a FA for the language

$$L = \{w \in \{a, b\}^* \mid w \text{ has at least three } a\text{'s or at least two } b\text{'s}\}.$$

Note that this language is a union of two languages.

**Task 2** Construct a FA for the language

$$L = \{w_1 w_2 \in \{0, 1\}^* \mid w_1 = 0^{2n}, w_2 = (100)^m, \text{ for some } n, m \in \mathbb{N}\}.$$

Note that the regular expression for  $L$  is  $(00)^* \cdot (100)^*$ .

**Task 3** Let  $L$  be the language of all strings over  $\{0, 1\}$  that do not contain a pair of 1's that are separated by an odd number of symbols. Give the state diagram of a DFA with 5 states that recognises  $L$ .

**Task 4** Construct a DFA for the language  $L^*$  where

$$L = 01 \cup (00)^* 11.$$

**Task 5** Let  $L$  be a regular language,  $L \subseteq \Sigma^*$ . Show that the reversed language of  $L$  defined as

$$L^R = \{w \in \Sigma^* \mid w^R \in L\}$$

where reversed words are defined inductively by

$$\varepsilon^R = \varepsilon, (ua)^R = au^R \text{ for } a \in \Sigma, u \in \Sigma^*$$

is regular as well.

Hint: From an automaton for  $L$ , construct an automaton for  $L^R$ .

**Task 6** Give state diagrams of NFAs with the specified number of states recognising each of the following languages. In all parts the alphabet is  $\{0, 1\}$  and the language is given via its regular expression.

- (a) The language 0 with two states.
- (b) The language  $0^*$  with one state.

- (c) The language  $(0 \cup 1)^*00$  with three states,
- (d) The language  $1^* \cdot (001^+)^*$  with three states.

**Task 7** Let  $\Sigma = \{0, 1\}$  and let

$$D = \{w \in \{0, 1\}^* \mid \#_{01}(w) = \#_{10}(w)\}.$$

Thus  $101 \in D$  because  $101$  contains a single  $10$  and a single  $01$ , but  $1010 \notin D$  because  $\#_{01}(1010) = 1$  but  $\#_{10}(1010) = 2$ .

Show that  $D$  is a regular language.

All the best for the rest of your studies!