

Formale Systeme Proseminar

Tasks for Week 13, 14.1.2020

Task 1 Show that for cardinals $|A|, |B|$ we have (as expected from the notation): $|A| \leq |B|$ iff $|B| \geq |A|$.

Task 2 Show that the function $f: \mathbb{Z} \rightarrow \mathbb{N}$ given by

$$f(k) = |k| = \begin{cases} k & \text{if } k \geq 0 \\ -k & \text{if } k < 0 \end{cases}$$

is a surjection. Hence, this, together with Task 1, proves that $|\mathbb{N}| \leq |\mathbb{Z}|$.

Task 3 Prove that $A \subseteq B \Rightarrow |A| \leq |B|$.

Task 4 Prove by induction that

$$\forall n \in \mathbb{N} \setminus \{0, 1\}. (1 + 3 + \dots + (2n - 1) = n^2).$$

Task 5 The sequence $(a_i \mid i \in \mathbb{N})$ is inductively defined by

$$a_0 = 0$$

$$a_{i+1} = a_i + 3$$

Prove (by induction) that $\forall n \in \mathbb{N}. 3|a_n$. Try to find a closed formula for a_n and prove by induction that it is really true.

Task 6 Prove by induction that if A is a finite set, i.e., $|A| = k$ for some $k \in \mathbb{N}$ then

$$|\mathcal{P}(A)| = 2^k.$$