

# Formale Systeme Proseminar

Tasks for Week 10, 10.12.2020

**Task 1** Let  $M = \{a, b, c\}$ . Give  $M \times M$ . Define (if possible) a relation  $R$  on  $M$  that is reflexive and symmetric, but not transitive.

**Task 2** Let  $M = \{a, b, c\}$ . Define (if possible) a relation  $R$  on  $M$  that is reflexive and transitive, but not symmetric.

**Task 3** Let  $M = \{a, b, c\}$ . Define (if possible) a relation  $R$  on  $M$  that is symmetric and transitive, but not reflexive.

**Task 4** Check if the following relation is reflexive, symmetric, and/or transitive:

$$R_1 = \{(x, y) \mid x, y \in \mathbb{R}, x = 0 \wedge y \geq 0\}.$$

**Task 5** Prove that for any set  $X$ , the diagonal relation  $\Delta_X = \{(x, x) \mid x \in X\}$  is reflexive, symmetric, and transitive.

**Task 6** For each of the following relations on  $\mathbb{N}$  find out if it is a partial order, a strict order, a preorder, a total order, or an equivalence:

- (a)  $xRy$  if and only if  $|x - y|$  is a multiple of 3.
- (b)  $xRy$  if and only if  $x < 10$  and  $y$  is even.

**Task 7** Is it possible that a relation  $R$  is both

- (a) symmetric and asymmetric?
- (b) symmetric and antisymmetric?

**Task 8** Prove that every strict order is antisymmetric.

**Task 9** Let  $X$  be a set. Consider the relation  $R$  on  $\mathcal{P}(X)$  defined by

$$(A, B) \in R \text{ iff } A \cap B = \emptyset.$$

Check if  $R$  is a partial order and/or an equivalence.