Formale Systeme Proseminar

Tasks for Week 6: 7.11.19

The first two tasks remained from last time.

Task 1 Show the following equivalences by calculating with propositions. Always state precisely: (1) which standard equivalence(s) you use, (2) whether you apply Substitution or Leibnitz, or both, and (3) how you do this.

   (a) $P \vee (P \wedge Q) \stackrel{val}{=} P$

   (b) $P \wedge (P \vee Q) \stackrel{val}{=} P$

   (c) $P \Rightarrow \neg Q \stackrel{val}{=} \neg(P \wedge Q)$

Task 2 Show with a calculation that the following formulas are tautologies

   (a) $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$

   (b) $P \vee \neg((P \Rightarrow Q) \Rightarrow P)$

Task 3 Show with calculations that for arbitrary sets A and B, we have $A \subseteq B$ if and only if $B^c \subseteq A^c$.

Task 4 Check with a calculation whether the following abstract propositions are equivalent:

   (a) $((a \Rightarrow b) \Rightarrow \neg a)$ and $(\neg b \vee \neg a) \wedge (\neg b \vee b)$

   (b) $a \wedge b$ and $(\neg a \vee b) \Leftrightarrow a$

Task 5 Prove with a calculation that

   (a) $(A^c)^c = A$ for any set A

   (b) $A \cup (A \cap B) = A$ for any two sets A and B.

Task 6 Check for every pair of propositions given below whether they are comparable (one is stronger than the other), or whether they are incomparable.

   (a) $P \vee Q$ and $P \wedge Q$

   (b) P and $\neg(P \vee Q)$

   (c) P and $\neg(P \Rightarrow Q)$

Task 7 Are the following statements valid? Why?

(a) If $P \overset{val}{\models} Q$ and $Q \overset{val}{\models} R$ and $R \overset{val}{\models} S$, then $P \overset{val}{\models} S$.

(b) If $P \overset{val}{\models} Q$ and $P \overset{val}{\models} R$, then $Q \overset{val}{=} R$.

(c) If $P \overset{val}{\models} Q$ and $P \overset{val}{\models} R$, then Q and R are incomparable.