Formale Systeme Proseminar

Tasks for Week 4, 24.10.2019

Task 1 Give the truth tables for each of the following abstract propositions:

(a) $(a \Rightarrow (b \Rightarrow a))$

(b) $((\neg(a \Rightarrow b)) \Leftrightarrow (a \wedge (\neg b)))$

(c) $((\neg(\neg a)) \Rightarrow ((\neg a) \wedge b))$

(d) $(a \Rightarrow ((b \wedge a) \vee c))$.

Task 2 For which values of a, b, and c one gets 0 in the truth-table of

$$(a \wedge (b \Rightarrow c)) \Rightarrow ((b \Rightarrow a) \wedge c) \ ?$$

Task 3 Check whether the following two propositions are equivalent:

(a) $\neg(b \vee \neg c)$ and $\neg b \wedge c$

(b) $a \Rightarrow b$ and $\neg a \Rightarrow \neg b$

(c) $(a \vee b) \wedge a$ and a

(d) $(a \vee b) \wedge b$ and $(b \wedge c) \vee (b \wedge \neg c)$.

Task 4 Give an example of a tautology (i.e., an abstract proposition that is always true independent of the truth-values of its variables) with only one proposition variable a and with only parentheses and

(a) connective $\Rightarrow$

(b) connectives $\vee$ and $\neg$

(c) connectives $\wedge$ and $\neg$

(d) connective $\Leftrightarrow$.

Task 5 Prove that:

(a) $P \Rightarrow Q$ is not equivalent to $Q \Rightarrow P$

(b) $P \Rightarrow Q$ is not equivalent to $\neg P \Rightarrow \neg Q$

(c) $P \Leftrightarrow Q \Leftrightarrow R$ is not equivalent to $(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R)$

Remember this!

Note that the remaining two tasks require calculating with propositions, which we will only learn on Wednesday, October 23 2019.

Task 6 Show the following equivalences by calculating with propositions. Always state precisely: (1) which standard equivalence(s) you use, (2) whether you apply Substitution or Leibnitz, or both, and (3) how you do this.

(a) $P \vee (\neg P \wedge Q) \overset{val}{=} P \vee Q$

(b) $P \wedge (P \Rightarrow Q) \overset{val}{=} P \wedge Q$

(c) $P \vee (P \wedge Q) \overset{val}{=} P$

(d) $P \wedge (P \vee Q) \overset{val}{=} P$

(e) $P \Rightarrow \neg Q \overset{val}{=} \neg(P \wedge Q)$

Task 7 Show with a calculation that the following formulas are tautologies

(a) $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$

(b) $P \vee \neg((P \Rightarrow Q) \Rightarrow P)$