

# Formale Systeme Proseminar

Tasks for Week 3, 17.10.2019

**Task 1** Prove that for any sets  $X$  and  $Y$ , we have  $X \cap Y \subseteq X$ .

**Task 2** Prove that for any set  $X$ , we have  $X \cup X = X$ .

**Task 3** Prove that for any set  $X$  there exist sets  $Y$  and  $Z$  such that  $X = Y \cup Z$ .

**Task 4** Prove that  $\emptyset \subseteq X$  for any set  $X$ .

**Task 5** Check in each of the following cases whether the given rule is correct. If it is, give arguments to show this. If it is not, give a counter example.

(a) There are K's which are also M's  
All K's are L's

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There are L's which are M's

(b) No one K is an M  
All K's are L's

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No one L is an M

**Task 6** For each of the following concrete propositions, write an abstract proposition which corresponds to it:

- (a) I love you and will always be true to you.
- (b) If it is raining, then I will stay home and watch a movie.
- (c)  $x^2 > 4$  if, and only if,  $x > 2$  or  $x < -2$ .
- (d) I will go climbing if you bring a rope.

**Task 7** Give the following propositions in words again, with 'it is raining' for  $a$ , 'it is windy' for  $b$ , and 'I am wet' for  $c$ .

- (a)  $a \wedge \neg b$
- (b)  $\neg(a \vee b)$
- (c)  $(a \Rightarrow c) \vee (b \Rightarrow \neg a)$ .
- (d)  $\neg\neg a$ .

**Task 8** Draw the trees of the following abstract propositions and give the main symbol for each of them.

(a)  $(a \Rightarrow (b \Rightarrow a))$

(b)  $((\neg(a \Rightarrow b)) \Leftrightarrow (a \wedge (\neg b)))$

(c)  $((\neg(\neg a)) \Rightarrow ((\neg a) \wedge b))$

(d)  $(a \Rightarrow ((b \wedge a) \vee c))$ .

**Task 9** Drop as many parentheses as possible from the abstract propositions of Task 4.