

Formale Systeme Proseminar

Tasks for Week 12, 19.12.2019

Task 1 Describe the equivalence classes of the equivalence \equiv_5 on \mathbb{Z} defined in the lectures. In general, for a fixed natural number n , describe the classes of \equiv_n . How many classes are there?

Task 2 Consider the relation $R \subseteq \mathbb{N} \times \mathbb{N}$ defined by

$$R = \{(n, n+1) \mid n \in \mathbb{N}\}.$$

- (a) Find the relation R^2 ,
- (b) Find the relation R^3 ,
- (c) Can you think of a concise way to describe the reflexive and transitive closure relation R^* ?

Task 3 Which of the following relations between $A = \{a, b, c\}$ and $B = \{1, 2\}$ are graphs of functions from A to B ?

- (a) $R_1 = \{(a, 1), (b, 2)\}$.
- (b) $R_2 = \{(a, 1), (b, 1), (b, 2), (c, 1)\}$.
- (c) $R_3 = \{(a, 1), (b, 2), (a, 2)\}$.
- (d) $R_4 = \{(a, 1), (b, 2), (c, 1)\}$.

Why?

Task 4 Let $X = \{1, 2, 3, 4, 5\}$ and consider the function $c: \mathcal{P}(X) \setminus \{\emptyset\} \rightarrow X$ defined by $c(Y) = |Y|$ for any $Y \subseteq X$, $Y \neq \emptyset$. Show that c is surjective but not injective.

Task 5 Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(n) = n + 5$ is an injection.

Task 6 Let X be any set. Show that the identity function $\text{id}_X: X \rightarrow X$ defined by $\text{id}_X(x) = x$ is a bijection.

Task 7 Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be two surjective functions. Prove that then $g \circ f$ is surjective as well.