Formale Systeme Proseminar

Tasks for Week 10, 5.12.2019

Task 1 Show with derivations that the following formula is a tautology

$$\exists_x \forall_y [P(x) \Rightarrow Q(y)] \Rightarrow (\forall_u [P(u)] \Rightarrow \exists_v [Q(v)])$$

Task 2 Prove with a derivation that the following formula is a tautology.

$$\exists_y [\forall_x [P(x) \wedge Q(x, y)]] \Rightarrow \forall_z [P(z)]$$

Task 3 Prove with a derivation that the following formula is a tautology.

$$\forall_y [Q(y) \Rightarrow (P(y) \Rightarrow \exists_x [P(x) \wedge Q(x)])]$$

Task 4 Prove with a derivation that the following formula is a tautology.

$$\forall_x [P(x) : Q(x)] \Rightarrow (\exists_x [P(x)] \Rightarrow \exists_x [Q(x)])$$

Also prove it with a calculation.

Task 5 Prove with a derivation that the following formula is a tautology.

$$\exists_x [\forall_y [P(x, y)]] \Rightarrow \forall_v [\exists_u [P(u, v)]]$$

Task 6 Let $M = \{a, b, c\}$. Give $M \times M$. Define (if possible) a relation R on M that is reflexive and symmetric, but not transitive.

Task 7 Let $M = \{a, b, c\}$. Define (if possible) a relation R on M that is reflexive and transitive, but not symmetric.

Task 8 Let $M = \{a, b, c\}$. Define (if possible) a relation R on M that is symmetric and transitive, but not reflexive.