

# Formale Systeme Proseminar

Tasks for Week 9, 29.11.2017

**Task 1** Show with derivations that the following formula is a tautology

$$\neg(P \Rightarrow Q) \Rightarrow \neg Q$$

**Task 2** Give logical derivation of the following tautology

$$(P \wedge \neg Q) \Rightarrow \neg(P \Rightarrow Q)$$

**Task 3** Give logical derivation of the following tautology

$$(\neg P \Rightarrow P) \Rightarrow P$$

**Task 4** Give logical derivation of the following tautology.

$$((P \Rightarrow Q) \Rightarrow \neg P) \Rightarrow (P \Rightarrow \neg Q)$$

**Task 5** Give logical derivation of the following tautology

$$(P \Rightarrow Q) \vee P$$

**Task 6** Give logical derivation of the case-distinction tautology:

$$(P \vee Q) \wedge (P \Rightarrow R) \wedge (Q \Rightarrow R) \Rightarrow R$$

**Task 7** Give a proof of the following proposition with the help of case distinction.

$$(x \geq 2 \vee x = -1) \Rightarrow x^3 - 3x - 2 \geq 0$$

for  $x \in \mathbb{R}$ .

Say precisely how you use the tautology

$$((P \vee Q) \wedge (P \Rightarrow R) \wedge (Q \Rightarrow R)) \Rightarrow R.$$