Formale Systeme Proseminar

Tasks for Week 6: 8.11.18

Task 1 Prove that:

(a) $P \Rightarrow Q$ is not equivalent to $Q \Rightarrow P$

(b) $P \Rightarrow Q$ is not equivalent to $\neg P \Rightarrow \neg Q$

(c) $P \Leftrightarrow Q \Leftrightarrow R$ is not equivalent to $(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R)$

Remember this!

Task 2 Show the following equivalences by calculating with propositions. Always state precisely: (1) which standard equivalence(s) you use, (2) whether you apply Substitution or Leibnitz, or both, and (3) how you do this.

(a) $P \vee (\neg P \wedge Q) \stackrel{val}{=} P \vee Q$

(b) $P \wedge (P \Rightarrow Q) \stackrel{val}{=} P \wedge Q$

(c) $P \vee (P \wedge Q) \stackrel{val}{=} P$

(d) $P \wedge (P \vee Q) \stackrel{val}{=} P$

(e) $P \Rightarrow \neg Q \stackrel{val}{=} \neg(P \wedge Q)$

Task 3 Show with a calculation that the following formulas are tautologies

(a) $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$

(b) $P \vee \neg((P \Rightarrow Q) \Rightarrow P)$

Task 4 Show with calculations that for arbitrary sets A and B, we have $A \subseteq B$ if and only if $B^c \subseteq A^c$.

Task 5 Check with a calculation whether the following abstract propositions are equivalent:

(a) $((a \Rightarrow b) \Rightarrow \neg a)$ and $(\neg b \vee \neg a) \wedge (\neg b \vee b)$

(b) $a \wedge b$ and $(\neg a \vee b) \Leftrightarrow a$

Task 6 Prove with a calculation that

(a) $(A^c)^c = A$ for any set A

(b) $A \cup (A \cap B) = A$ for any two sets A and B.

The material for the following two tasks will only be taught on Wednesday October 25!

Task 7 Check for every pair of propositions given below whether they are comparable (one is stronger than the other), or whether they are incomparable.

(a) $P \vee Q$ and $P \wedge Q$

(b) P and $\neg(P \vee Q)$

(c) P and $\neg(P \Rightarrow Q)$

Task 8 Are the following statements valid? Why?

(a) If $P \overset{val}{\models} Q$ and $Q \overset{val}{\models} R$ and $R \overset{val}{\models} S$, then $P \overset{val}{\models} S$.

(b) If $P \overset{val}{\models} Q$ and $P \overset{val}{\models} R$, then $Q \overset{val}{=} R$.

(c) If $P \overset{val}{\models} Q$ and $P \overset{val}{\models} R$, then Q and R are incomparable.