Formale Systeme Proseminar

Tasks for Week 8, 23.11.2017

Task 1 Show with a calculation that

(a) $\exists x[P : Q] \stackrel{val}{=} \neg \forall x[Q : \neg P]$,

(b) $\forall x[P : Q \vee R] \stackrel{val}{=} \forall x[P \wedge \neg Q : R]$.

Task 2 Is the following statement true? If yes, prove it with a calculation; if not, give a counter example.

$$\neg \exists_x[P : Q] \stackrel{val}{=} \forall_x[Q : P]$$

Task 3 Show with derivations that the following formula is a tautology

$$((P \Rightarrow Q) \Rightarrow P) \Rightarrow ((P \Rightarrow Q) \Rightarrow Q)$$

Task 4 For each of the line numbers of your solution to Task 6, say where the proposition which occurs on that line is valid (i.e. allowed to be used).

Task 5 Give logical derivation of the following tautology.

$$(P \wedge (Q \Rightarrow R)) \Rightarrow ((P \Rightarrow Q) \Rightarrow (P \wedge R))$$

Task 6 Show with derivations that the following formula is a tautology

$$\neg(P \Rightarrow Q) \Rightarrow \neg Q$$

Task 7 Give logical derivation of the following tautology

$$(P \wedge \neg Q) \Rightarrow \neg(P \Rightarrow Q)$$