Formale Systeme Proseminar

Tasks for Week 7, 16.11.2017

Task 1 Write the following statements as formulas with quantifiers. D is a subset of $\mathbb{N}$.

(a) All elements of D are larger than or equal to 0.

(b) All elements of D are larger than 5 and less than 15.

(c) All elements of D are larger than 5 or all elements of D are smaller than 15.

(d) Every pair of different elements of D differ by at least 2.

Task 2 Write the following statements as formulas with quantifiers.

(a) For every natural number, there is a natural number which is greater than it by 5.

(b) There is no natural number which is greater than all natural numbers.

(c) There are two natural numbers the sum of whose squares is 40.

(d) The sum of two natural numbers is greater than or equal to each of the two numbers.

Are the propositions true? Give an explanation.

Task 3 Is the following proposition true?

$$\forall x\ [x \in \mathbb{Z} : \exists y\ [y \in \mathbb{Z} : x + y = 0]] \Rightarrow \exists y\ [y \in \mathbb{Z} : \forall x\ [x \in \mathbb{Z} : x + y = 0]]$$

Explain your answer.

Task 4 Check which of the following propositions are equivalent independently of D where D is an arbitrary subset of $\mathbb{R}$.

(a) $\exists x\ [x \in D : \forall y\ [y \in D : y \geq x]]$

(b) $\exists l\ [l \in D : \forall k\ [k \in D : l \leq k]]$

(c) $\exists k\ [k \in D : \forall m\ [m \in D : \neg(k < m)]]$

(d) $\forall y\ [y \in D : \exists x\ [x \in D : y \leq x]]$

Task 5 Show with a counter example that the following properties hold.

(a) $\forall x[P : Q] \overset{val}{\neq} \forall x[Q : P]$

(b) $\exists x[P : Q] \wedge \exists x[P : R] \overset{val}{\neq} \exists x[P : Q \wedge R]$

Task 6 Is the following statement always true? Why?

$$\forall x[A(x) : B(x)] \Rightarrow \exists x[B(x)]$$