Formale Systeme Proseminar

Tasks for Week 13, 11.1.2018

Task 1 Show that the function $f\colon \mathbb{N} \to \mathbb{N}$ given by $f(n) = n + 5$ is an injection.

Task 2 Show that the function $f\colon \mathbb{Z} \to \mathbb{N}$ given by

$$f(k) = |k| = \begin{cases} k & \text{if } k \geq 0 \\ -k & \text{if } k < 0 \end{cases}$$

is a surjection.

Task 3 Let $f\colon A \to B$ and $g\colon B \to C$ be two injective functions. Prove that then $g \circ f$ is injective as well. (Hence, you need to prove Lemma I4 from the lectures.)

Task 4 Prove by induction that

$$\forall n \in \mathbb{N} \setminus \{0, 1\}. (1 + 3 + \ldots + (2n - 1) = n^2).$$

Task 5 The sequence $(a_i \mid i \in \mathbb{N})$ is inductively defined by

$a_0 = 0$

$a_{i+1} = a_i + 3$

Prove (by induction) that $\forall n \in \mathbb{N}. 3 | a_n$. Try to find a closed formula for a_n and prove by induction that it is really true.

Task 6 Prove that $A \subseteq B \Rightarrow |A| \leq |B|$.
(Note: You need to construct an injection from A to B.)

Task 7 Prove by induction that if A is a finite set, i.e., $|A| = k$ for some $k \in \mathbb{N}$ then

$$|\mathcal{P}(A)| = 2^k.$$

Task 8 Prove that for any set X, $|\mathcal{P}(X)| = 2^{|X|}$, i.e., provide a bijection from $\mathcal{P}(X)$ to the set $\{0, 1\}^X$ of all functions from X to $\{0, 1\}$.