

Formale Systeme Proseminar

Tasks for Week 10, 7.12.2017

Task 1 Prove with a derivation that the following formula is a tautology.

$$\forall x[P(x) : Q(x)] \Rightarrow (\exists x[P(x)] \Rightarrow \exists x[Q(x)])$$

Also prove it with a calculation.

Task 2 Prove with a derivation that the following formula is a tautology.

$$\exists x[\forall y[P(x, y)]] \Rightarrow \forall v[\exists u[P(u, v)]]$$

Task 3 Let $M = \{a, b, c\}$. Give $M \times M$. Define (if possible) a relation R on M that is reflexive and symmetric, but not transitive.

Task 4 Let $M = \{a, b, c\}$. Define (if possible) a relation R on M that is reflexive and transitive, but not symmetric.

Task 5 Let $M = \{a, b, c\}$. Define (if possible) a relation R on M that is symmetric and transitive, but not reflexive.

Task 6 Check if the following relation is reflexive, symmetric, and/or transitive:

$$R_1 = \{(x, y) \mid x, y \in \mathbb{R}, x = 0 \wedge y \geq 0\}.$$

Task 7 Is it possible that a relation R is both

- (a) symmetric and asymmetric?
- (b) symmetric and antisymmetric?

Task 8 Prove that every asymmetric relation is irreflexive as well.