Formale Systeme Proseminar

Tasks for Week 8

Task 1 Give logical derivation of the following tautology

$$(P \Rightarrow Q) \vee P$$

Task 2 Investigate whether the following formula is a tautology. If so, give a derivation to prove this; if not so, give a counterexample.

$$(P \Rightarrow Q) \Rightarrow (P \vee (Q \Rightarrow R))$$

Task 3 Give a proof of the following proposition with the help of case distinction.

$$(x \geq 2 \vee x = -1) \Rightarrow x^3 - 3x - 2 \geq 0$$

for $x \in \mathbb{R}$.

Say precisely how you use the tautology

$$((P \vee Q) \wedge (P \Rightarrow R) \wedge (Q \Rightarrow R)) \Rightarrow R.$$

Task 4 Show with derivations that the following formula is a tautology

$$\exists_x \forall_y [P(x) \Rightarrow Q(y)] \Rightarrow (\forall_u [P(u)] \Rightarrow \exists_v [Q(v)])$$

Task 5 Prove with a derivation that the following formula is a tautology.

$$\exists_y [\forall_x [P(x) \wedge Q(x, y)]] \Rightarrow \forall_z [P(z)]$$

Task 6 Prove with a derivation that the following formula is a tautology.

$$\forall_x [P(x) : Q(x)] \Rightarrow (\exists_x [P(x)] \Rightarrow \exists_x [Q(x)])$$

Also prove it with a calculation.

Task 7 Prove with a derivation that the following formula is a tautology.

$$\exists_x [\forall_y [P(x, y)]] \Rightarrow \forall_v [\exists_u [P(u, v)]]$$