Formale Systeme Proseminar

Tasks for Week 14, 21.1.2016

Task 1 Prove that for any set X, $|\mathcal{P}(X)| = 2^{|X|}$, i.e., provide a bijection from $\mathcal{P}(X)$ to the set $\{0,1\}^X$ of all functions from X to $\{0,1\}$.

Task 2 Prove that $\aleph_0 \cdot 2 = \aleph_0$, i.e., prove that $\mathbb{N} \times \{0,1\}$ is a countable set (a set with cardinality equal to the cardinality of $\mathbb{N}$).

Task 3 Construct a DFA for the language

$$L = \{w \in \{0,1\}^* \mid w \text{ begins with a 1 and ends with a 0}\}.$$

Task 4 Construct a DFA for the language

$$L = \{w \in \{a,b\}^* \mid \text{ every } a \text{ in } w \text{ is preceeded and followed by a } b\}.$$

Task 5 Construct a DFA for the language

$$L = \{w \in \{0,1,2,3,4,5,6,7,8,9\}^* \mid w \text{ as a natural number is divisible by 3}\}.$$

Hint: A natural number is divisible by 3 iff the sum of its digits is divisible by 3.

Task 6 Construct a DFA for the language

$$L = \{w \in \{0,1\}^* \mid w \text{ ends with 11 or with 101}\}.$$

Task 7 Let L be the language of all strings over $\{0,1\}$ that do not contain a pair of 1's that are separated by an odd number of symbols. Give the state diagram of a DFA with 5 states that recognizes L.