Formale Systeme Proseminar

Tasks for Week 6

Task 1 Let $A = \{a, b, c, d\}$. For each of the following partitions of A write down the corresponding equivalence:

(a) $\{\{a, b\}, \{c, d\}\}$,

(b) $\{\{a\}, \{b, c, d\}\}$,

(c) $\{\{a\}, \{b\}, \{c\}, \{d\}\}$.

Task 2 Give an example of an equivalence on $\mathbb{N}$ with

(a) 3 equivalence classes,

(b) 10 equivalence classes,

(c) 100 equivalence classes.

Task 3 Consider the relation $R \subseteq \mathbb{N} \times \mathbb{N}$ defined by

$$R = \{(n, n + 1) \mid n \in \mathbb{N}\}.$$

(a) Find the relation R^2,

(b) Find the relation R^3,

(c) Can you think of a concise way to describe the reflexive and transitive closure relation R^*?

Task 4 Which of the following relations between $A = \{a, b, c\}$ and $B = \{1, 2\}$ define (are graphs of) functions from A to B?

(a) $R_1 = \{(a, 1), (b, 2)\}$.

(b) $R_2 = \{(a, 1), (b, 1), (b, 2), (c, 1)\}$.

(c) $R_3 = \{(a, 1), (b, 2), (a, 2)\}$.

(d) $R_4 = \{(a, 1), (b, 2), (c, 1)\}$.

Why?

Task 5 Let $A = \{a, b, c\}$ and $B = \{1, 2\}$. Give an example of a function $f: A \to B$ with the property that every element $b \in B$ is the image under f of at least one element of A.

Task 6 Give an example of a function $f: \mathbb{N} \to \mathbb{N}$ with the property that no two elements are mapped to the same element.

Task 7 Let $X = \{1, 2, 3, 4, 5\}$ and consider the function $c: \mathcal{P}(X) \setminus \{\emptyset\} \to X$ defined by $c(Y) = |Y|$ for any $Y \subseteq X$, $Y \neq \emptyset$. Show that (1) every element of X is an image of at least one element of $\mathcal{P}(X) \setminus \emptyset$ and (2) there are two elemenst in $\mathcal{P}(X) \setminus \emptyset$ that are mapped to the same element of X.

Task 8 Let X be any set and R an equivalence relation on X. We denote by X/R the quotient set of R-equivalence classes (the partition corresponding to R), i.e.,

$$X/R = \{[x]_R \mid x \in X\}.$$

Does the assignement $[x]_R \mapsto x$ define a function from X/R to X? Why?