Formale Systeme Proseminar

Tasks for Week 14

As for some of you cardinals may be confusing and difficult, to each task I added "(Note: ...)" which recalls some of the definitions and clarifies what you should really do. I hope this is of help. The actual tasks are as given without the "(Note: ...)" part.

Task 1 Prove by induction that

$$\forall n[n \in \mathbb{N} \wedge n > 1 : 1 + 3 + \ldots + (2n - 1) = n^2].$$

Task 2 The sequence $(a_i \mid i \in \mathbb{N})$ is inductively defined by

$a_0 = 0$
$a_{i+1} = a_i + 3$

Prove (by induction) that $\forall n[n \in \mathbb{N} : 3|a_n]$. Try to find a closed formula for a_n and prove by induction that it is really true.

Task 3 The sequence $(a_i \mid i \in \mathbb{N})$ is inductively defined by

$a_0 = 1$
$a_{i+1} = \frac{1}{i+1} \sum_{k=0}^{i} a_k$

Prove (by induction) that $\forall n[n \in \mathbb{N} : a_n = 1]$.

Task 4 Consider the sets $A = \{a, b\}$ and $B = \{1, 2, 3\}$. Convince yourself on this very example that $|A^B| = |A|^{|B|}$, i.e., that $|A^B| = 8$.

Task 5 Prove that $A \subseteq B \Rightarrow |A| \leq |B|$.
(Note: You need to construct an injection from A to B.)

Task 6 Prove that any subset of a finite set is finite.
(Note: you need to show that if A is a finite set, i.e., there is a bijection $f\colon A \to \mathbb{N}_k$ for some $k \in \mathbb{N}$ and $B \subseteq A$, then there is a bijection $g\colon B \to \mathbb{N}_m$ for some $m \in \mathbb{N}$.)

Task 7 Let $A_{m,n} = \{k \in \mathbb{N} \mid n \leq k \leq m\}$. Prove that $A_{m,n}$ is a finite set, by explicitly defining a bijection (to one of the sets $\mathbb{N}_l$ for some $l \in \mathbb{N}$).