Formale Systeme Proseminar

Tasks for Week 12

Task 1 Give logical derivation of the following tautology

$$(P \Rightarrow Q) \vee P$$

Task 2 Investigate whether the following formula is a tautology. If so, give a derivation to prove this; if not so, give a counterexample.

$$(P \Rightarrow Q) \Rightarrow (P \vee (Q \Rightarrow R))$$

Task 3 Give a proof of the following proposition with the help of case distinction.

$$(x \geq 2 \vee x = -1) \Rightarrow x^3 - 3x - 2 \geq 0$$

for $x \in \mathbb{R}$.

Say precisely how you use the tautology

$$((P \vee Q) \wedge (P \Rightarrow R) \wedge (Q \Rightarrow R)) \Rightarrow R.$$

Task 4 Give logical derivation of the following tautology.

$$(P \wedge (Q \Rightarrow R)) \Leftrightarrow ((P \Rightarrow Q) \Rightarrow (P \wedge R))$$

Task 5 Give logical derivation of the following tautology.

$$((P \Rightarrow Q) \Rightarrow \neg P) \Rightarrow (P \Rightarrow \neg Q)$$

Task 6 Prove with a derivation that the following formula is a tautology.

$$\forall_y [Q(y) \Rightarrow (P(y) \Rightarrow \exists_x [P(x) \wedge Q(x)])]$$

Task 7 Prove with a derivation that the following formula is a tautology.

$$\forall_x [P(x) : Q(x)] \Rightarrow (\exists_x [P(x)] \Rightarrow \exists_x [Q(x)])$$

Also prove it with a calculation.