

Formale Systeme Proseminar

Tasks for Week 8

Task 1 Prove that:

- (a) $P \Rightarrow Q$ is not equivalent to $Q \Rightarrow P$
- (b) $P \Rightarrow Q$ is not equivalent to $\neg P \Rightarrow \neg Q$
- (c) $P \Leftrightarrow Q \Leftrightarrow R$ is not equivalent to $(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R)$

Remember this!

Task 2 In a mathematics book we read

$$y = x^2 + 2x + 2 \Rightarrow y = (x + 1)^2 + 1 \Rightarrow y \geq 1.$$

What is meant here? Are the arrows, implication arrows?

Task 3 Show that Contraposition also holds for \Leftrightarrow , that is:

$$P \Leftrightarrow Q \stackrel{val}{=} \neg Q \Leftrightarrow \neg P.$$

Task 4 Show the following equivalences by calculating with propositions. Always state precisely: (1) which standard equivalence(s) you use, (2) whether you apply Substitution or Leibnitz, or both, and (3) how you do this.

- (a) $P \Rightarrow Q \stackrel{val}{=} (P \wedge Q) \Leftrightarrow P$
- (b) $P \wedge (P \vee Q) \stackrel{val}{=} P$
- (c) $P \vee (P \wedge Q) \stackrel{val}{=} P$
- (d) $P \wedge (P \Rightarrow Q) \stackrel{val}{=} P \wedge Q$
- (e) $P \vee (\neg P \wedge Q) \stackrel{val}{=} P \vee Q$

Task 5 Show with a calculation that the following formulas are tautologies

- (a) $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$
- (b) $P \vee \neg((P \Rightarrow Q) \Rightarrow P)$

Task 6 Show with calculations that $0 < x^2 - 2x + 1 < 9$ is equivalent to

$$x \neq 1 \wedge -2 < x < 4.$$

Task 7 Check for every pair of propositions given below whether they are comparable (one is stronger than the other), or whether they are incomparable.

- (a) $P \vee Q$ and $P \wedge Q$
- (b) P and $\neg(P \vee Q)$
- (c) P and $\neg(P \Rightarrow Q)$

Task 8 Are the following statements valid? Why?

- (a) If $P \stackrel{val}{\models} Q$ and $Q \stackrel{val}{\models} R$ and $R \stackrel{val}{\models} S$, then $P \stackrel{val}{\models} S$.
- (b) If $P \stackrel{val}{\models} Q$ and $P \stackrel{val}{\models} R$, then $Q \stackrel{val}{=} R$.
- (c) If $P \stackrel{val}{\models} Q$ and $P \stackrel{val}{\models} R$, then Q and R are incomparable.

Task 9 Show with a calculation:

- (a) $P \Rightarrow Q \stackrel{val}{\models} (P \wedge R) \Rightarrow (Q \wedge R)$
- (b) $\neg(P \Rightarrow \neg Q) \stackrel{val}{\models} (P \vee R) \wedge Q$