Formale Systeme Proseminar

Tasks for Week 5

Task 1 Let $A = \{1, 2, 3, 4\}$ and consider the relation

$$R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1), (3,4), (4,3)\}.$$

(a) Show that R is an equivalence relation.

(b) What are the equivalence classes of R?

Task 2 Consider the relation $R \subseteq \mathbb{Z} \times \mathbb{Z}$ given as

$$R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid \ (xy > 0) \text{ or } x = y = 0\}.$$

Prove that R is an equivalence and write down the equivalence classes of R (the quotient $\mathbb{Z}/R$).

Task 3 Show that the relation **on** $\mathbb{N} \times \mathbb{N}$ defined by

$$(a, b)R(c, d) \text{ if and only if } a + d = b + c$$

is an equivalence.

Task 4 Let $A = \{a, b, c, d\}$. For each of the following partitions of A write down the corresponding equivalence:

(a) $\{\{a, b\}, \{c, d\}\}$,

(b) $\{\{a\}, \{b, c, d\}\}$,

(c) $\{\{a\}, \{b\}, \{c\}, \{d\}\}$.

Task 5 Prove that the relation $\bigtriangledown_X = X \times X$ is an equivalence relation for any set X. How many classes does $\bigtriangledown_X$ have? What is the quotient $X/\bigtriangledown_X$?

Task 6 For each of the following relations on $\mathbb{N}$ find out if it is a partial order, a strict order, a preorder, a total order, or an equivalence:

(a) xRy if and only if $x \leq y$,

(b) xRy if and only if $x < y$,

(c) xRy if and only if $|x - y|$ is a multiple of 5.

(d) xRy if and only if $x < 10$ and y is even.

Task 7 Let $A = \{a, b, c\}$. How many equivalence relations are there on A? List them all.

(Hint: Think of partitions.)