Formale Systeme Proseminar

Tasks for Week 4

Task 1 Let $A = \{1, 2, 3, 4\}$, $B = \{1, 4\}$ and $C = \{1\}$.

(a) Give $B \times A$, $B \times B$, and $B \times C$.

(b) What is $B \times \emptyset$?

(c) How many elements does $A \times B \times C$ have?

Task 2 Prove that if $A \cup B = \emptyset$, then $A \cap B = \emptyset$.

Task 3 Check whether the following propositions always hold. If so, give a proof; if not, give a counterexample.

(a) If $A \subseteq B$, then $A^c \subseteq B^c$.

(b) If $A \subseteq B$, then $B^c \subseteq A^c$.

Task 4 Check whether the following propositions always hold. If so, give a proof; if not, give a counterexample.

(a) $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$

(b) $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$

Task 5 Check whether each of the following relations is reflexive, symmetric, and/or transitive:

(a) $R_1 = \{(n, m) \mid n, m \in \mathbb{N} \text{ and } n > 0 \text{ and } m > 0\}$

(b) $R_2 = \{(u, v) \mid u, v \in A^* \text{ and } u \text{ is a prefix of v}\}$

Task 6 Let $M = \{1, 2, 3\}$. Give $M \times M$. Define a relation R on M that is:

(a) a partial order.

(b) an equivalence.

(c) transitive and symmetric, but not reflexive.

Task 7 Let X be a set. Show that the diagonal relation

$$\Delta_X = \{(x, x) \mid x \in X\}$$

is both an equivalence relation and a partial order relation. It is the only such relation on X.

Task 8 Let $X = \{1, 2, 3, 4, 5\}$. Can you write an algorithm that checks whether a relation R on X is reflexive, symmetric, and/or transitive? What data structure would you use to represent the input relation R?