Formale Systeme Proseminar

Tasks for Week 11

Task 1 Show with derivations that the following formula is a tautology

$$\exists_x \forall_y [P(x) \Rightarrow Q(y)] \Rightarrow (\forall_u [P(u)] \Rightarrow \exists_v [Q(v)])$$

Task 2 Prove with a derivation that the following formula is a tautology.

$$\exists_y [\forall_x [P(x) \wedge Q(x,y)]] \Rightarrow \forall_z [P(z)]$$

Task 3 Prove with a derivation that the following formula is a tautology.

$$\forall_y [Q(y) \Rightarrow (P(y) \Rightarrow \exists_x [P(x) \wedge Q(x)])]$$

Task 4 Prove with a derivation that the following formula is a tautology.

$$\forall_x [P(x) : Q(x)] \Rightarrow (\exists_x [P(x)] \Rightarrow \exists_x [Q(x)])$$

Task 5 Prove with a derivation that the following formula is a tautology.

$$\exists_x [\forall_y [P(x,y)]] \Rightarrow \forall_v [\exists_u [P(u,v)]]$$

Task 6 Prove the set property

$$A \subseteq B \Rightarrow B^c \subseteq A^c$$

normally and with derivations (in any order), where A and B are arbitrary sets.

Task 7 Prove the set property

$$(A \cap B) \cup B = B$$

with a calculation.

Task 8 Prove the set property

$$A \subseteq B \Rightarrow A \cap B = A$$

normally and with derivations (in any order), where A and B are arbitrary sets.

Task 9 Let V be a set and R a relation on V which is symmetric and transitive. Prove:

$$\forall x [x \in V \colon \exists y [y \in V \colon R(x,y)]] \Rightarrow (R \text{ is an equivalence relation }).$$

Write a normal proof and a flag proof.