Formale Systeme Proseminar

Tasks for Week 10

Task 1 Show with derivations that the following formula is a tautology

$$((P \Rightarrow Q) \Rightarrow P) \Rightarrow ((P \Rightarrow Q) \Rightarrow Q)$$

Task 2 For each of the line numbers of your solution to Task 1, say where the proposition which occurs on that line is valid (i.e. allowed to be used).

Task 3 Show with derivations that the following formula is a tautology

$$\neg(P \Rightarrow Q) \Rightarrow \neg Q$$

Task 4 Give logical derivation of the following tautology

$$(P \wedge \neg Q) \Rightarrow \neg(P \Rightarrow Q)$$

Task 5 Give logical derivation of the following tautology

$$(\neg P \Rightarrow P) \Rightarrow P$$

Task 6 Give logical derivation of the following tautology

$$(P \Rightarrow Q) \vee P$$

Task 7 Investigate whether the following formula is a tautology. If so, give a derivation to prove this; if not so, give a counterexample.

$$(P \Rightarrow Q) \Rightarrow (P \vee (Q \Rightarrow R))$$

Task 8 Give a proof of the following proposition with the help of case distinction.

$$(x \geq 2 \vee x = -1) \Rightarrow x^3 - 3x - 2 \geq 0$$

for $x \in \mathbb{R}$.

Say precisely how you use the tautology

$$((P \vee Q) \wedge (P \Rightarrow R) \wedge (Q \Rightarrow R)) \Rightarrow R.$$

Task 9 Give logical derivation of the following tautology.

$$(P \wedge (Q \Rightarrow R)) \Leftrightarrow ((P \Rightarrow Q) \Rightarrow (P \wedge R))$$

Task 10 Give logical derivation of the following tautology.

$$((P \Rightarrow Q) \Rightarrow \neg P) \Rightarrow (P \Rightarrow \neg Q)$$