

# Formale Systeme

Test 1, Group 1, 13.12.2019

**Task 1.** (25 points) Prove that the following propositional formula is a tautology in at least two different ways (via a truth table, or via a calculation, or via a derivation):

$$\neg(P \wedge Q) \wedge Q \Rightarrow \neg P$$

**Task 2.** (20 points) Show that the following formula is not a tautology:

$$(P \Rightarrow R) \Rightarrow ((P \vee Q) \Rightarrow (Q \wedge R))$$

**Task 3.** (15 points) Let  $\mathbb{P}$  denote the set of all prime (natural) numbers. Write the following statement as a predicate formula:

*Every prime number between 10 and 100 equals the sum of two natural numbers that are not prime.*

**Task 4.** (25 points) Write an abstract predicate formula corresponding to the following syllogism example, and prove that it is a tautology.

No birds can ski.  
All penguins are birds.

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Therefore, no penguins can ski.

**Task 5.** (20 points) Prove that for arbitrary sets  $A$  and  $B$ ,

$$(A \setminus B) \cup A = A.$$