Finite Automata

Alphabets and Languages

Def

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Σ - alphabet (finite set)

$\Sigma^n = \{a_1a_2..a_n \mid a_i \in \Sigma\}$ is the set of words of length n

$\Sigma^* = \{w \mid \exists n \in \mathbb{N}.\ \exists\ a_1, a_2, .., a_n \in \Sigma.\ w = a_1a_2..a_n\}$ is the set of all words over Σ

Alphabets and Languages

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$\Sigma^0 = \{\varepsilon\}$ contains only the empty word

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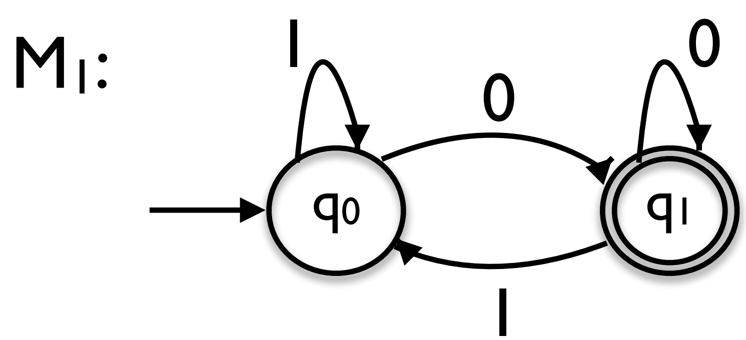
A language L over Σ is a subset $L \subseteq \Sigma^*$

Deterministic Automata (DFA)

Informal example

$\Sigma = \{0,1\}$



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alphabet

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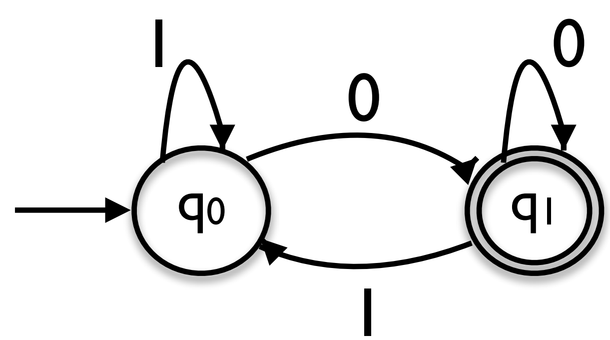

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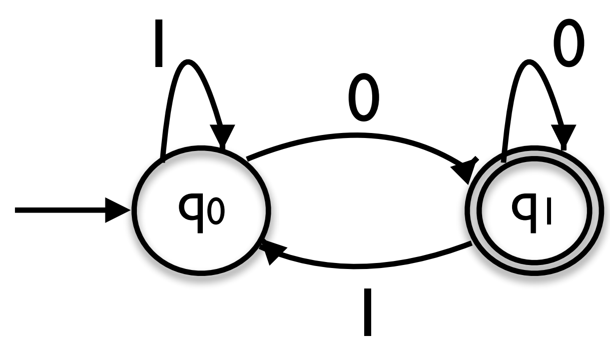

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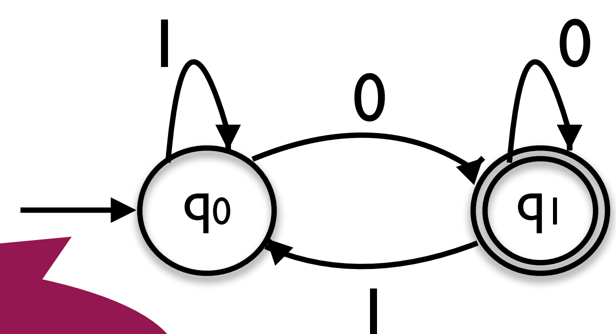

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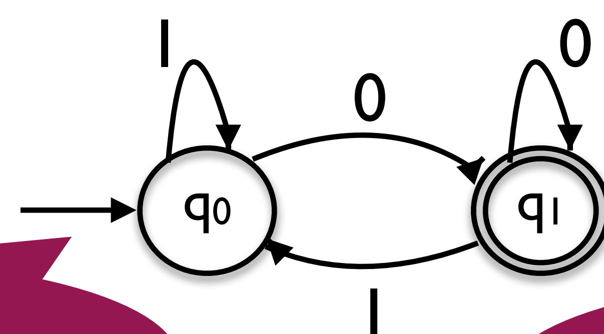

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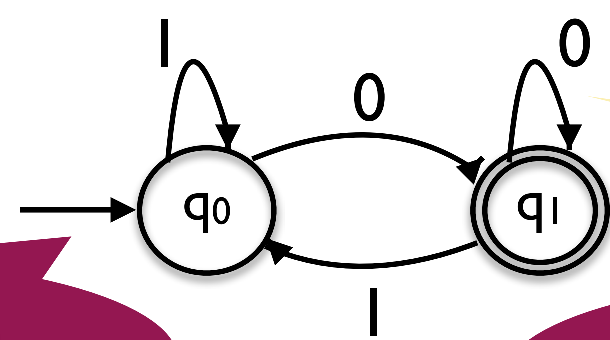

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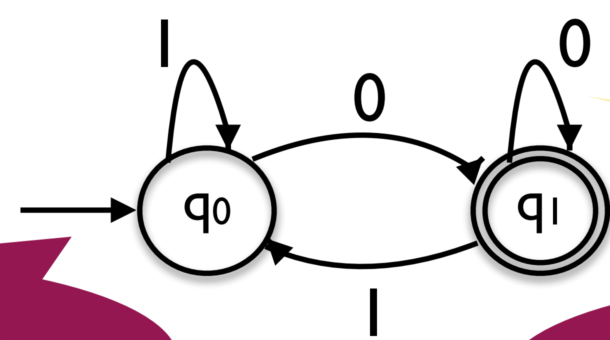

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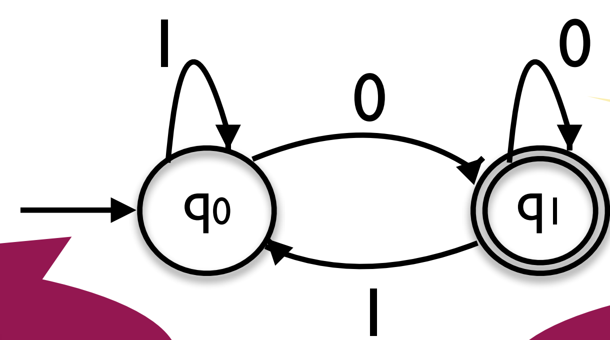

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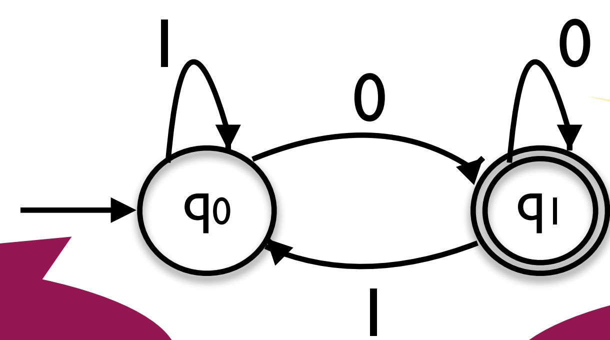

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regular language

regular expression

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$\delta(q_0, 0) = q_1$,$\delta(q_0, 1) = q_0$
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DFA

The extended transition function

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$$L(M) = \{w \in \Sigma^* \mid \delta^*(q_0, w) \in F\}$$

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$L(M_1) = \{w0 | w \in \{0,1\}^*\}$

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Regular languages and operations

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Let Σ be an alphabet. A language L over Σ ($L \subseteq \Sigma^*$) is regular iff it is recognised by a DFA.

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Regular operations

Let L, L_1, L_2 be languages over Σ. Then
$L_1 \cup L_2$, $L_1 \cdot L_2$, and L^* are languages, where

$$L_1 \cdot L_2 = \{w_1 \cdot w_2 \mid w_1 \in L_1, w_2 \in L_2\}$$

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$\varepsilon \in L^*$ always

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Regular expressions

Definition

finite representation of infinite languages

Regular expressions

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Regular expressions

Definition

Let Σ be an alphabet. The following are regular expressions

1. a for $a \in \Sigma$
2. ε
3. $\varnothing$
4. $(R_1 \cup R_2)$ for R_1, R_2 regular expressions
5. $(R_1 \cdot R_2)$ for R_1, R_2 regular expressions
6. $(R_1)^*$ for R_1 regular expression

finite representation of infinite languages

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inductive

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corresponding languages

$L(a) = \{a\}$
$L(\varepsilon) = \{\varepsilon\}$
$L(\emptyset) = \emptyset$
$L(R_1 \cup R_2) = L(R_1) \cup L(R_2)$
$L(R_1 \cdot R_2) = L(R_1) \cdot L(R_2)$
$L(R_1^*) = L(R_1)^*$

Closure under regular operations

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Theorem C1

The class of regular languages is closed under union

Closure under regular operations

also under intersection

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The class of regular languages is closed under union

Theorem C2

The class of regular languages is closed under complement

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Theorem C1

The class of regular languages is closed under union

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The class of regular languages is closed under complement

Theorem C3

The class of regular languages is closed under concatenation

Closure under regular operations

also under intersection

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The class of regular languages is closed under union

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Theorem C4

The class of regular languages is closed under Kleene star

Closure under regular operations

also under intersection

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We can already prove these!

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But not yet these two…

Theorem C4

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Equivalence of regular expressions and regular languages

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Theorem (Kleene)

A language is regular (i.e., recognised by a finite automaton) iff it is the language of a regular expression.

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needs nondeterminism

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Nondeterministic Automata (NFA)

Informal example

$\Sigma = \{0,1\}$

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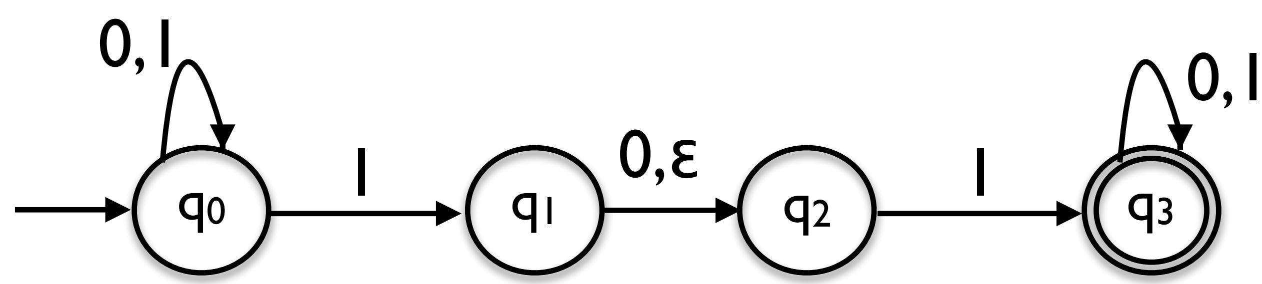

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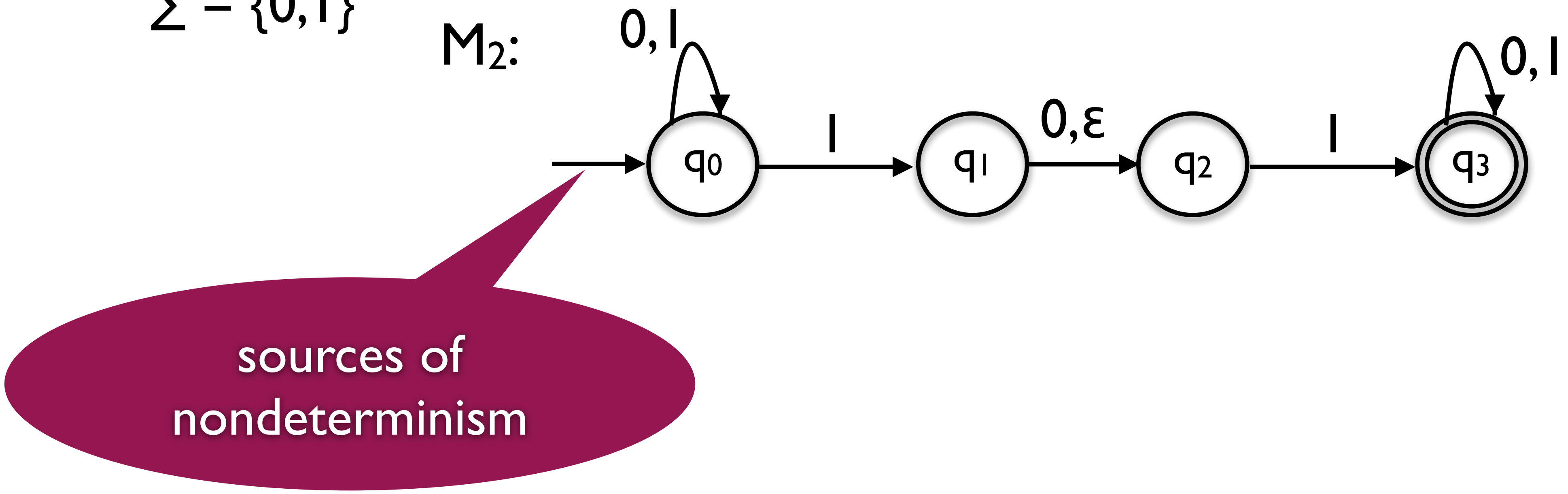

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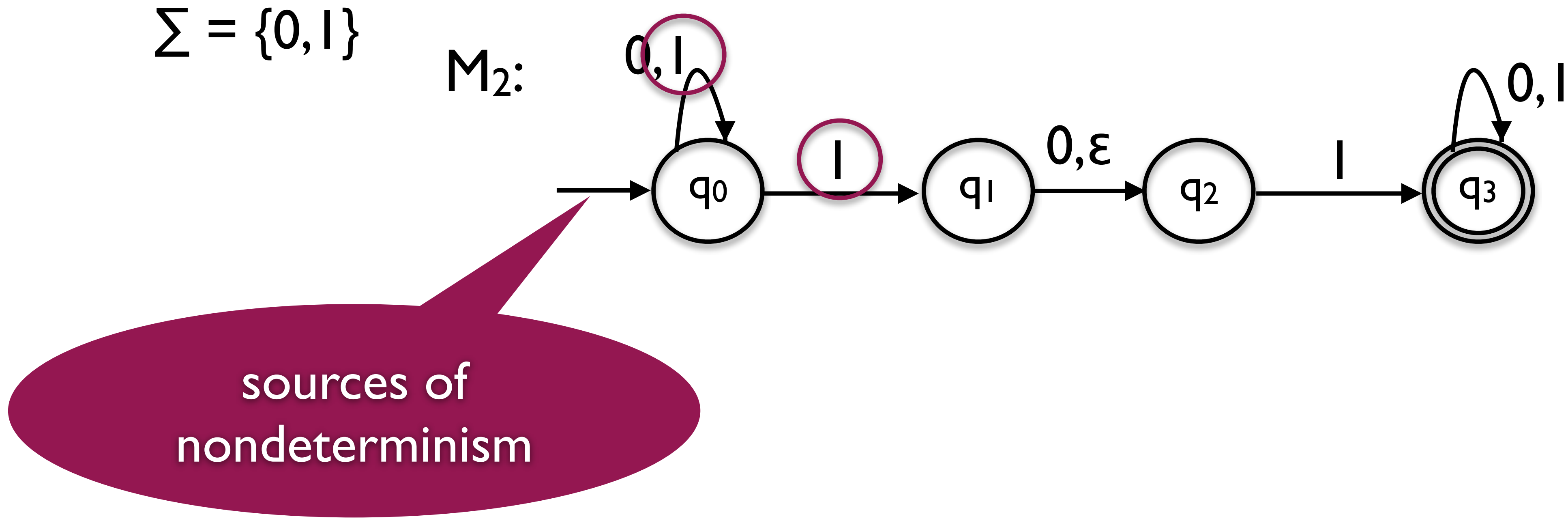

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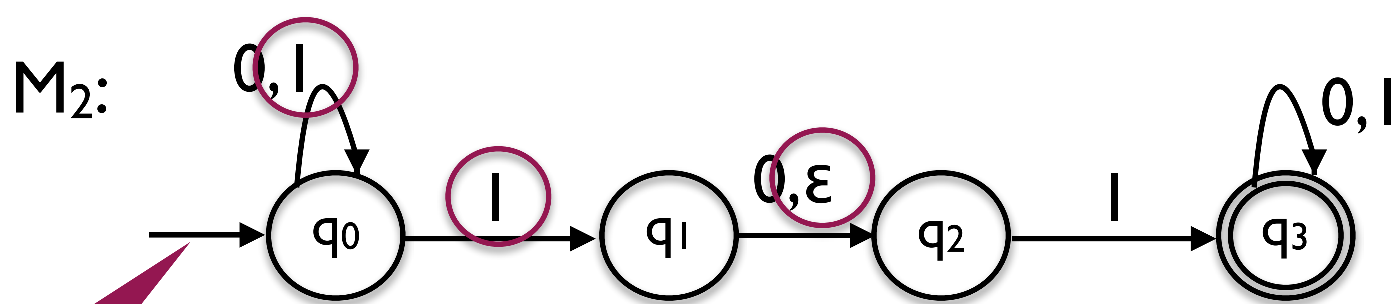

sources of nondeterminism

Nondeterministic Automata (NFA)

no 1 transition

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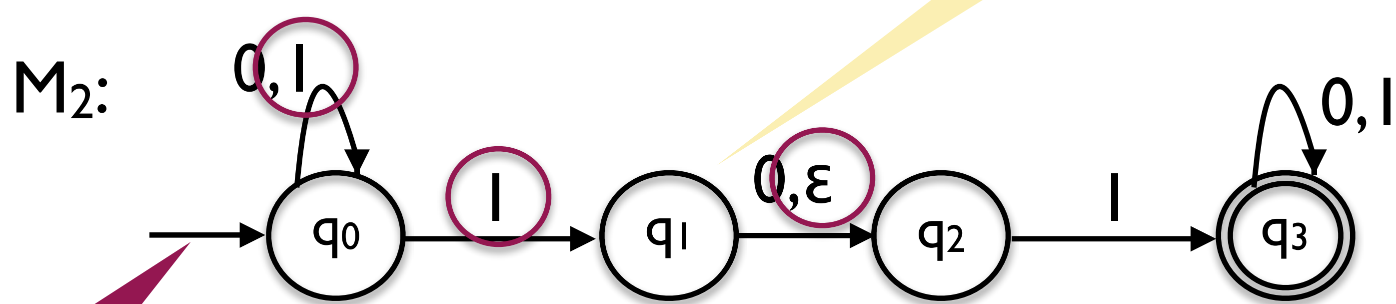

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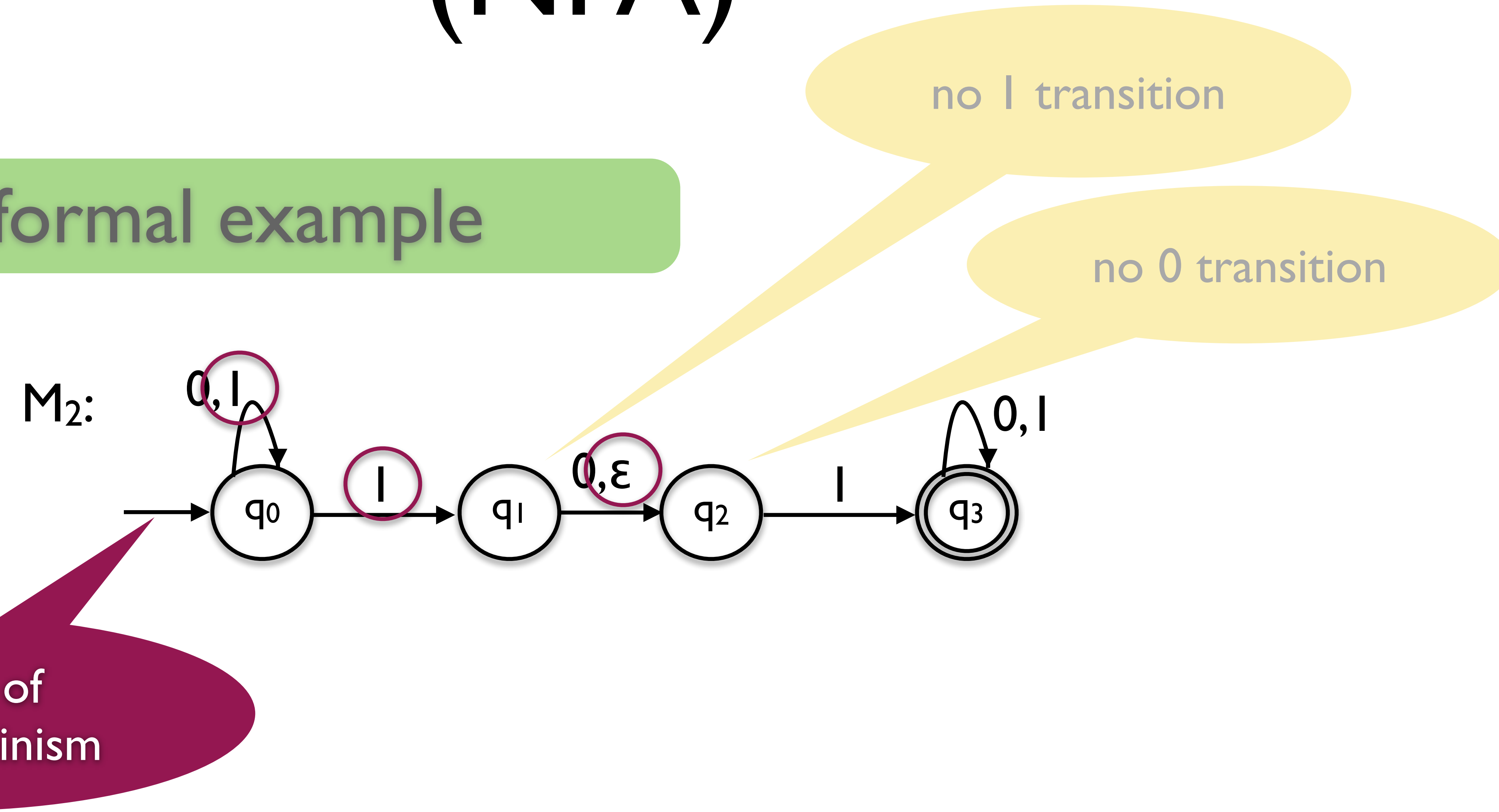

Nondeterministic Automata (NFA)

no 1 transition

Informal example

no 0 transition

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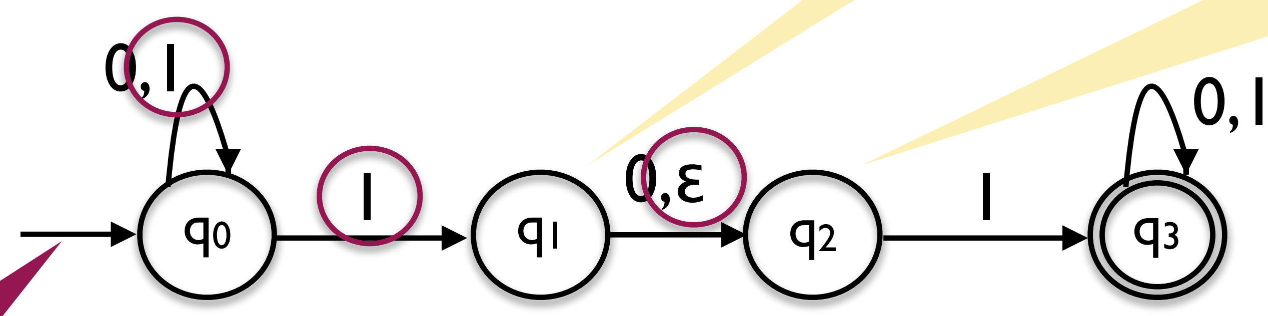
M_2:



sources of nondeterminism

Accepts a word iff there exists an accepting run

NFA

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$\delta(q_0, 0) = \{q_0\}$
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$\delta(q_0, \varepsilon) = \varnothing$
.....

NFA

The extended transition function

NFA

The extended transition function

Given an NFA $M = (Q, \Sigma, \delta, q_0, F)$ we can extend $\delta: Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$ to

$\delta^*: Q \times \Sigma^* \longrightarrow \mathcal{P}(Q)$

inductively, by:

$\delta^*(q, \varepsilon) = E(q)$ and $\delta^*(q,wa) = E(\bigcup_{q' \in \delta^*(q,w)} \delta(q', a))$

NFA

E(q) is the ε-closure of q, all states reachable by ε-transitions from q

The extended transition function

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inductively, by:

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The language recognised / accepted by a nondeterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ is

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Corollary

A language is regular iff it is recognised by a NFA

Closure under regular operations

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Theorem C1

The class of regular languages is closed under union

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Closure under regular operations

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Now we can prove these too

Theorem C4

The class of regular languages is closed under Kleene star

Nonregular languages

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Theorem (Pumping Lemma)

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every long enough word of a
regular language can be pumped

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If L is a regular language, then there is a number $p \in \mathbb{N}$ (the pumping length) such that for any $w \in L$ with $|w| \geq p$, there exist $x, y, z \in \Sigma^*$ such that $w = xyz$ and

1. $xy^iz \in L$, for all $i \in \mathbb{N}$
2. $|y| > 0$
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Note the logical structure!