LOGIC and SETS

Formale Systeme

3VO + 2PS

Lecturer: Ana Sokolova

Instructions: Ana Sokolova + Sebastian Arming

http://cs.uni-salzburg.at/~anas/FormaleSysteme2018/

The Rules of the Game

- **Lectures** Wednesday 2 pm - 2:45 pm in T01
  Thursday 10:15 am - 12 am in T01

- **Instructions**
  Group 1, Thursday 1:15 pm - 3 pm (AS) in T01
  Group 2, Thursday 3:15 pm - 5 pm (AS) in T01
  Group 3, Thursday 1:15 pm - 3 pm (SA) in T03

- **Tutors** Simon Bauer and Philipp Mayer
  Tuesday 11 am - 1 pm in T06

starting in two weeks

- **Books**
  Logical Reasoning: A First Course by R. Nederpelt and F. Kamaraddine

  How to Think Like a Mathematician by K. Houston

  Modellierung: Grundlagen und formale Methoden by U. Kastens and H. Kleine Büning

  Introduction to Automata Theory, Languages, and Computation by J. E. Hopcroft, R. Motwani and J.D. Ullman

The Rules... Instructions (PS)

- Instruction exercises on the web
  http://cs.uni-salzburg.at/~anas/FormaleSystemeProseminar2018/
  on Thursday afternoons
- To be solved by the students (ideally alone)
- In class we will have a small test every week except the first week (1 simple exercise)
  and then present solutions/discuss the exercises
  (sometimes students will be asked to present)

The Rules... Instructions (PS)

- The test exercise will be graded each week
- The graded exercise will be returned to you in class (with feedback) one week later
- Grade based on
  (1) the grades of the test exercises and
  (2) activity in class (ability to present solutions)
- All information about the course / rules / exams / grading is / will be on the course webpage

The Rules... Exam (VO)

- Written exams
- Written exam in February, April, and July
  or two partial tests during the semester
- Grade based on the # of points on the written exam (or sum of the points on the partial tests)
- For better grade oral exam after the written one upon appointment
- You can pass the course if you have 55% of the maximal points on the exam.

The Rules... Tests (VO)

- One test end of November, one beginning of February
- The tests are partial (half material)
- You can pass via tests if the sum of your points on both tests is at least 55% of the sum of maximal points on the tests and if on each test you have at least 20% of the maximal points
- The tests and the exams consist of exercises / questions related to the material taught in class
- You need not register for the tests, just be there

Some advice

- It starts easy, but soon it gets more difficult
- There accumulates lots of material for the exam
- Best is to regularly study, practice, solve the exercises yourself!

ECTS points

1 ECTS = 25 hours

- 3 ECTS for the lectures (VO)
- 4 ECTS for the instructions (PS)

7 x 25 = 175 hours

in class app. 50 hours

125 hours remain for your studying !

better: 1 week before exam (40 hours) + 7 hours a PS week

app, 10 hours a PS week :-)

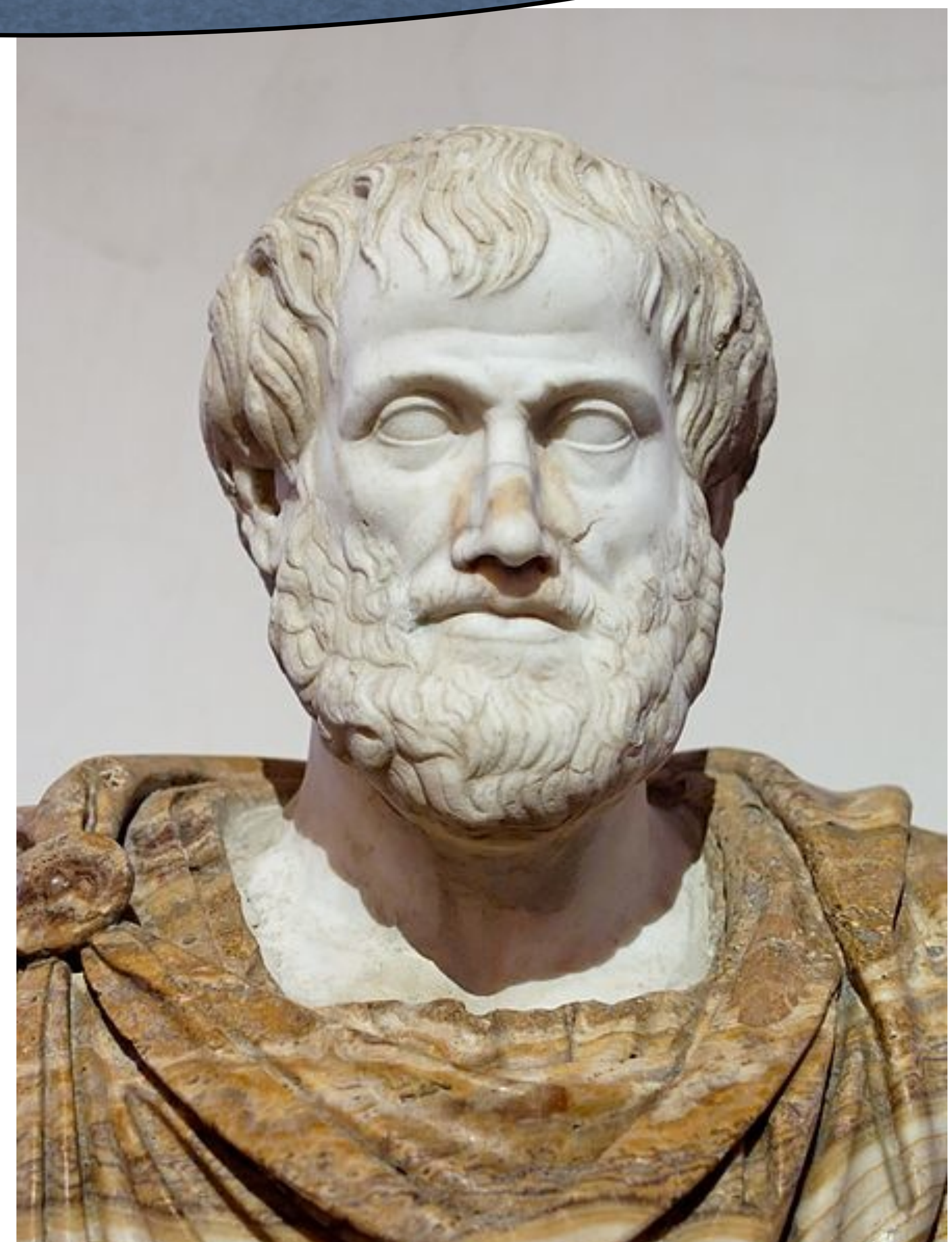
Logic = study of correct reasoning

In the beginning

Aristotle +/- 350 B.C.

Organon

19 syllogisms



Formal Logic

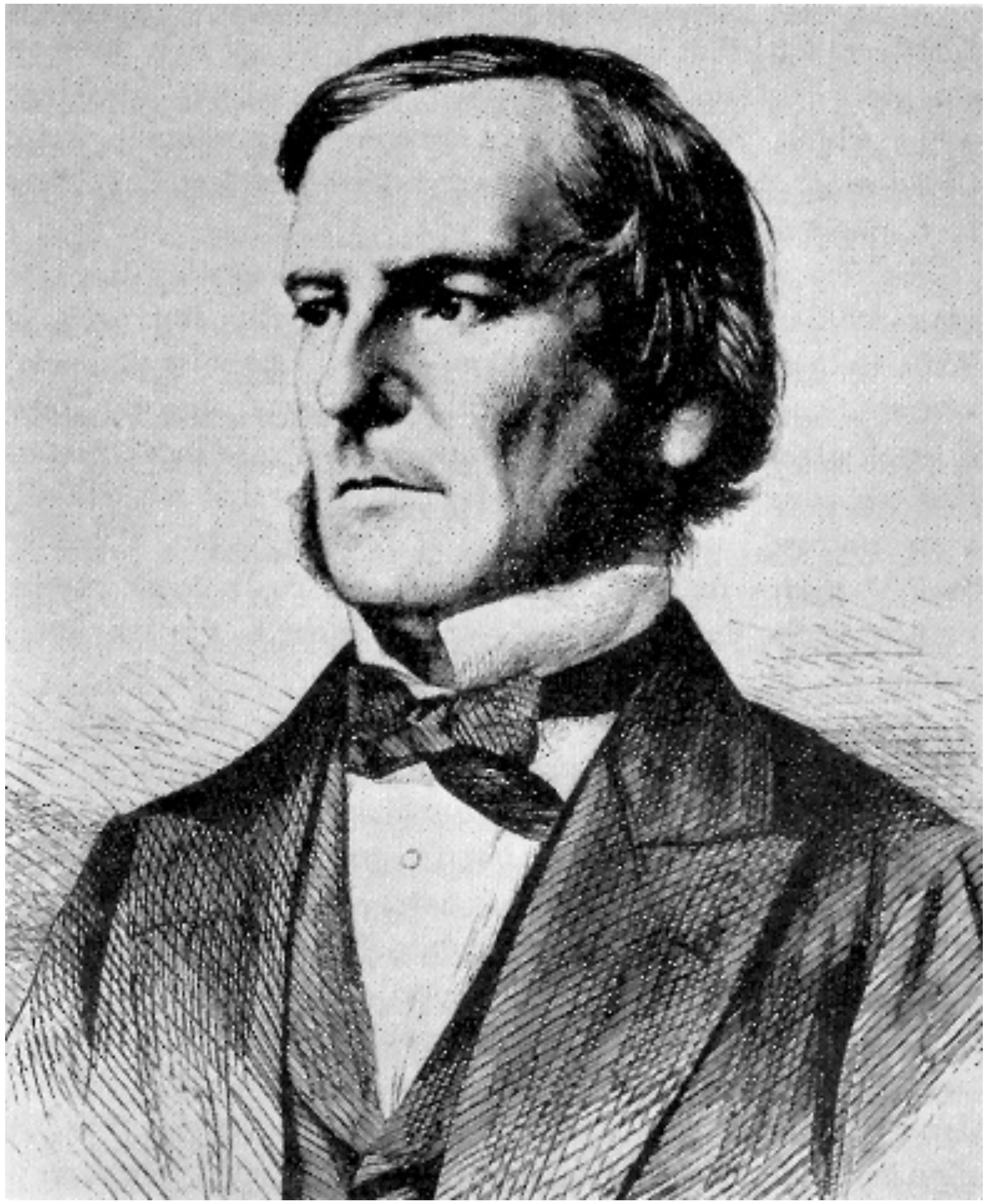
Gottfried Wilhelm Leibnitz
(1646 - 1716)

Beginnings of symbolic logic

Boolean Logic

George Boole
(1815 - 1864)

Boolean logic



We will learn

Starting this week

- Naive Set Theory - sets, relations, mappings, numbers and structures, ordered sets
- Logical Calculations - propositional logic, predicate logic
- Logical Derivations - reasoning
- Basics of formal models - finite automata, transition systems, graphs, grammars...

and understand !

Goals:

At the end of the course,
you will be able to:

- Read - formal mathematical statements, proofs
- Write - formal mathematical statements
- Manipulate - logical formulas
  - basic notions of set theory
- Prove - properties in basic set theory,
  - tautologies
- Argue about validity and truth
- Understand, construct, reason about - basic models of computation (finite automata)

Why formal models/ methods?

- For showing correctness of solutions, software and hardware
- For better understanding of a complex system, problem, task,... models, abstractions are needed
- For rigorous precise reasoning about a complex system, problem, task
- For automation

The river-crossing puzzle

- A man stands with a wolf, a goat, and a cabbage at the left bank of a river, that he wants to cross.
- The man has a boat that is large enough to carry him and another object to the other side.
- If the man leaves the wolf and the goat, or the goat and the cabbage on one side without supervision, one of them will get eaten :-(
- Is it possible to cross the river so that neither the goat nor the cabbage is eaten?

[Hopcroft et al, Kastens et al]

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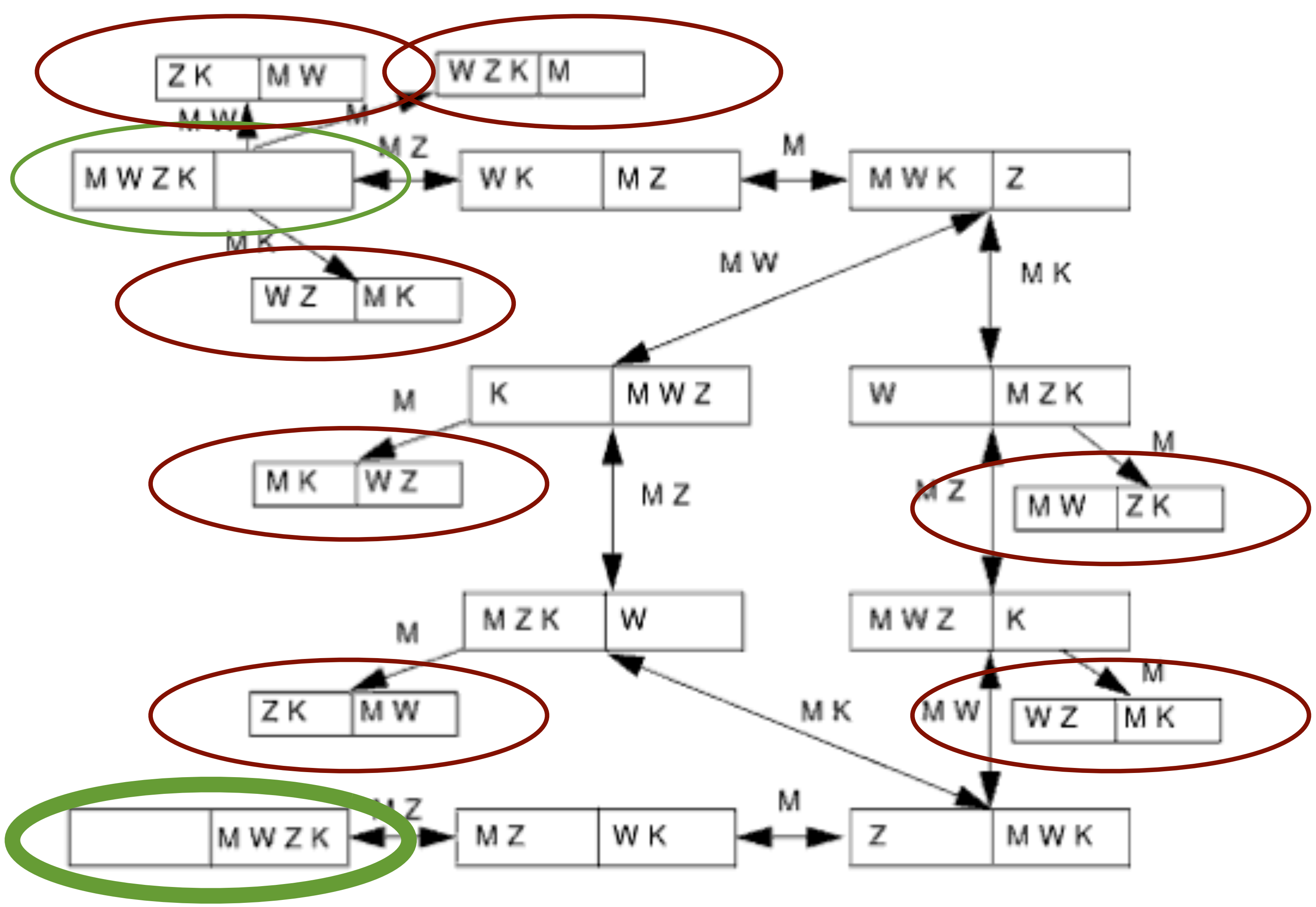
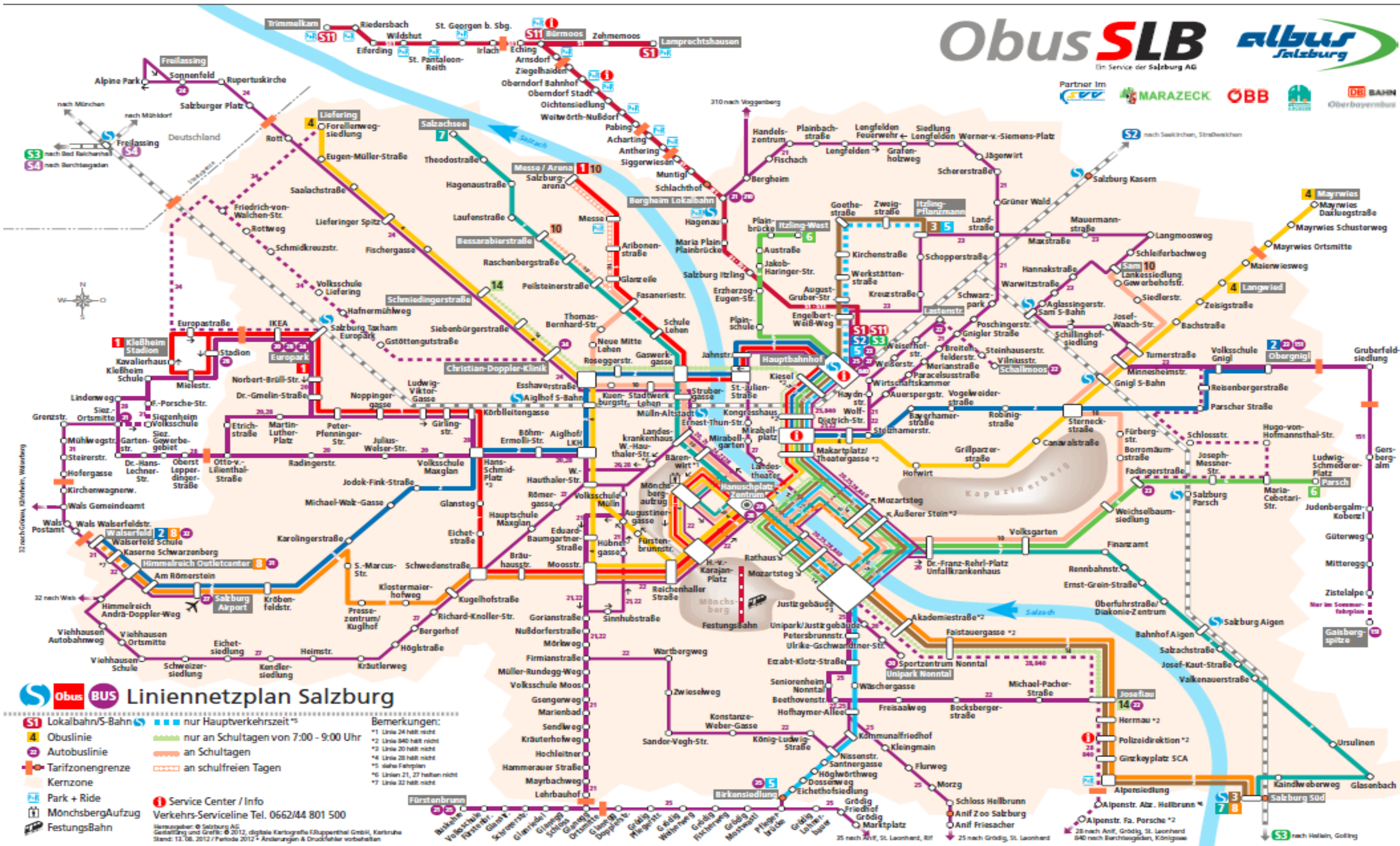
The river-crossing puzzle

Formalization with a finite automaton [Kastens et al.] :



states and transitions

Another model example



Puzzles 1

The Camels

Four tasmanian camels traveling on a very narrow ledge encounter four tasmanian camels coming the other way.

As everyone knows, tasmanian camels never go backwards, especially when on a precarious ledge. The camels will climb over each other, but only if there is a camel sized space on the other side.

The camels didn't see each other until there was only exactly one camel's width between the two groups.

How can all camels pass, allowing both groups to go on their way, without any camel reversing?

Two Strings

You have two strings whose only known property is that when you light one end of either string it takes exactly one hour to burn. The rate at which the strings will burn is completely random and each string is different.

How do you measure 45 minutes?

Puzzles 2

The Socks

Cathy has twelve black socks and twelve white socks in her drawer.

In complete darkness, and without looking, how many socks must she take from the drawer in order to be sure to get a pair that match?

The Pot of Beans

A pot contains 75 white beans and 150 black ones. Next to the pot is a large pile of black beans.

A somewhat demented cook removes the beans from the pot, one at a time, according to the following strange rule: He removes two beans from the pot at random. If at least one of the beans is black, he places it on the bean-pile and drops the other bean, no matter what color, back in the pot. If both beans are white, on the other hand, he discards both of them and removes one black bean from the pile and drops it in the pot.

At each turn of this procedure, the pot has one less bean in it. Eventually, just one bean is left in the pot. What color is it?

Puzzles 3

The Double Jeopardy Doors

You are trapped in a room with two doors. One leads to certain death and the other leads to freedom. You don't know which is which.

There are two robots guarding the doors. They will let you choose one door but upon doing so you must go through it.

You can, however, ask one robot one question. The problem is one robot always tells the truth, the other always lies and you don't know which is which.

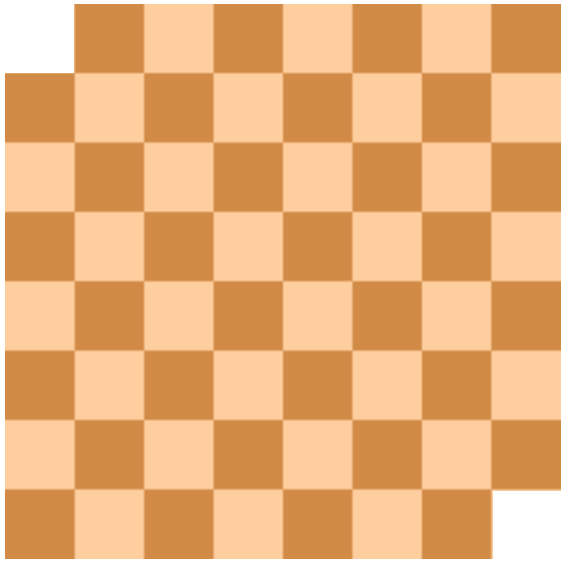
What is the question you ask?

The Mutilated Chessboard

We are give a chessboard and 31 dominos.
Suppose we cut off two squares at diagonally opposite corners of the board.

Is it possible to place the 31 dominos on the board so that all squares are covered?

If so, show how it can be done. If not, prove it impossible.



Puzzles 4

The Cannibals

Three cannibals and three anthropologists have to cross a river.

The boat they have is only big enough for two people. The cannibals will do as requested, even if they are on the other side of the river, with one exception. If at any point in time there are more cannibals on one side of the river than anthropologists, the cannibals will eat them.

What plan can the anthropologists use for crossing the river so they don't get eaten?

Note: One anthropologist can not control two cannibals on land, nor can one anthropologist on land control two cannibals on the boat if they are all on the same side of the river. This means an anthropologist will not survive being rowed across the river by a cannibal if there is one cannibal on the other side.

The MU puzzle

You are given a starting string, MI, and must obtain MU (or show that it is impossible) by using any combination of the following four rules:

- If you have a string ending in I, you can obtain that string with U appended. So if you have MI you can get MIU, and if you have MUII you can get MUIIU.
- If you have a string Mx, where x is any string, you can obtain Mxx. So if you have MI you can get MII, and if you have MUM you can get MUMUM.
- If you have a string containing three consecutive I's, you can obtain the string that replaces them with a single U. So if you have MIIIU you can get MUU, and if you have MUIIIMU, you can get MUUMU.
- If you have a string containing two consecutive U's, you can obtain the string that deletes them. So if you have MUUIU, you can get MIU, and if you have MUMUU you can get MUM.

Puzzles 5

The Boxes

There are three boxes. One is labeled "APPLES" another is labeled "ORANGES". The last one is labeled "APPLES AND ORANGES". You know that each is labeled incorrectly. You may ask me to pick one fruit from one box which you choose.

How can you label the boxes correctly?

The Prisoners and the Light Switch

100 prisoners are sentenced to life in prison in solitary confinement. Upon arrival at the prison, the warden proposes a deal to keep them entertained, certain that the prisoners are too dim-witted and impatient to accomplish it.

The warden has a large bowl containing the cell numbers of all the prisoners. Each day he randomly chooses one cell from the bowl, the corresponding prisoner is taken to the interrogation room, and the cell number is returned to the bowl.

While in the interrogation room, the prisoner will not be allowed to touch anything except the light switch, which the prisoner may choose to turn on or off.

The prisoner may make the assertion that all 100 prisoners have been in the room. If the prisoner’s assertion is correct, all prisoners will be released. If the prisoner is incorrect, the game is over and their chance to be freed is gone.

The prisoners are given one meeting to discuss a strategy before their communication is completely severed.
What strategy should they adopt in order to ensure, with 100% certainty, that one of them will guess correctly and all be freed?

The initial state of the light is OFF when the first prisoner enters the room.

Puzzles 6

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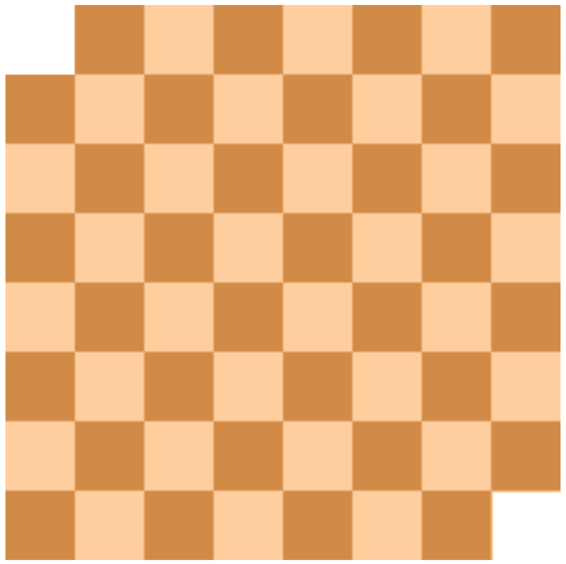

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