

Logic

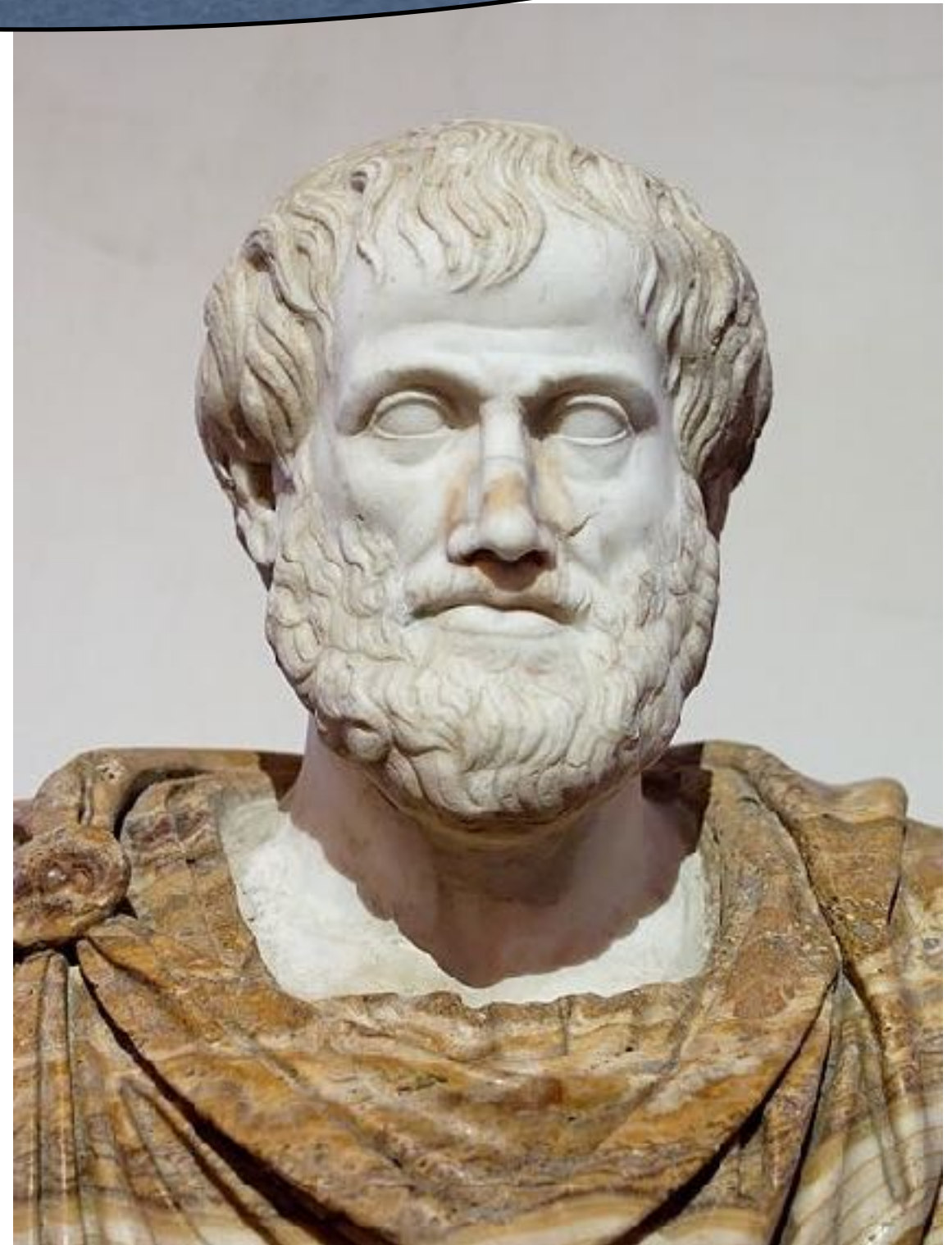
Logic = study of correct reasoning

In the beginning

Aristotle +/- 350 B.C.

Organon

19 syllogisms



Barbara syllogism

only later called so,
in the Middle Ages

All K's are L's
All L's are M's

All K's are M's

from the two
premises

one can
always conclude the
conclusion

independent of what the parameters K,L,M are

Logic (Logos, Greek for word, understanding, reason) deals with general reasoning laws in the form of formulas with parameters.

Propositions

Def. A proposition (**Aussage**) is a grammatically correct sentence that is either true or false.

Connectives

- \wedge for “and”
- \vee for “or”
- \neg for “not”
- \Rightarrow for “if .. then” or “implies”
- \Leftrightarrow for “if and only if”

logic deals with patterns!
what matters are not particular propositions but the way in which (abstract) propositions are combined and what follows from them

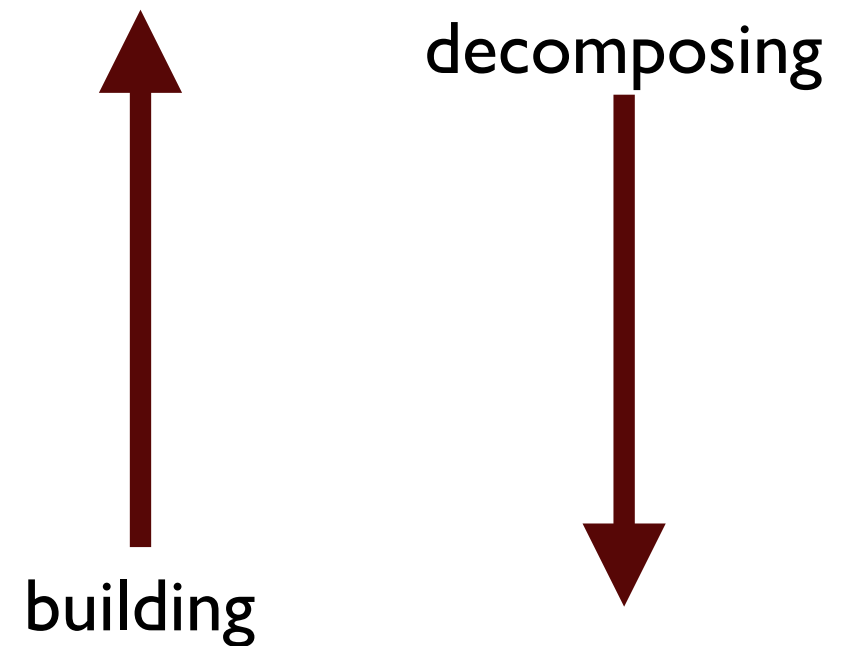
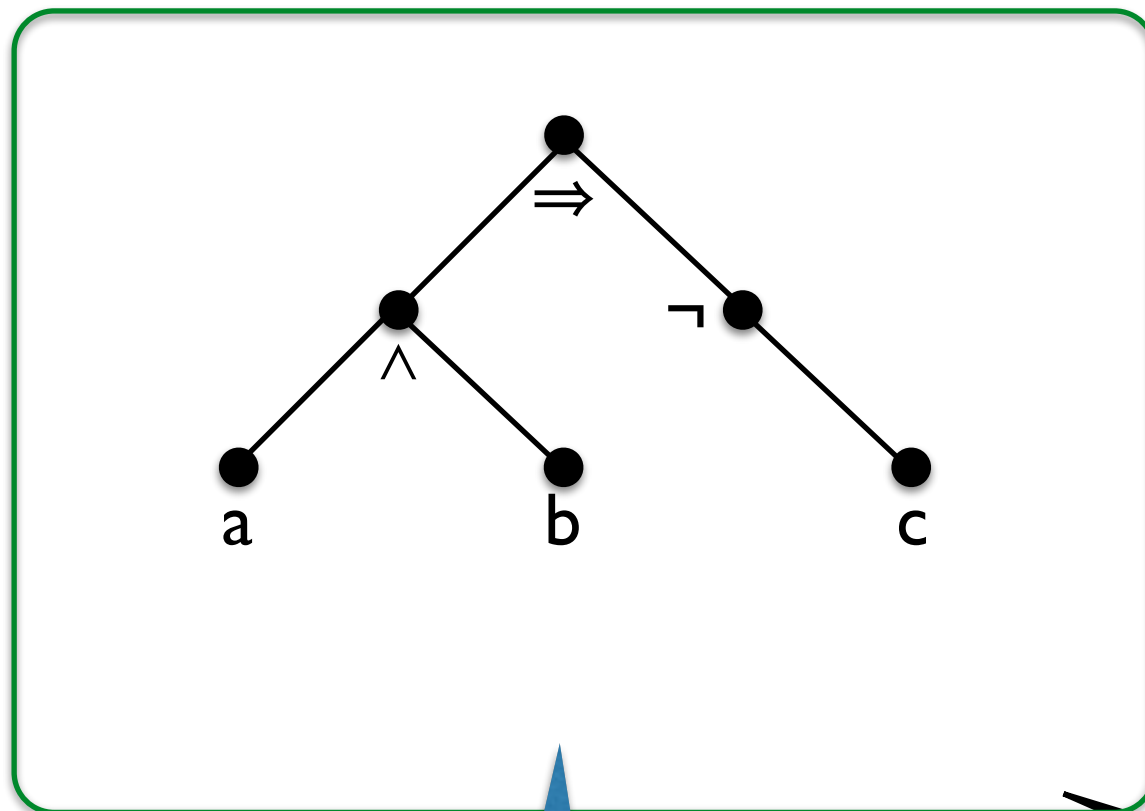
Abstract propositions

Definition

- Basis** Propositional variables are abstract propositions.
- Step (Case 1)** If P is an abstract proposition, then so is $(\neg P)$.
- Step (Case 2)** If P and Q are abstract propositions, then so are $(P \wedge Q)$, $(P \vee Q)$, $(P \Rightarrow Q)$, and $(P \Leftrightarrow Q)$.

a recursive/inductive
definition

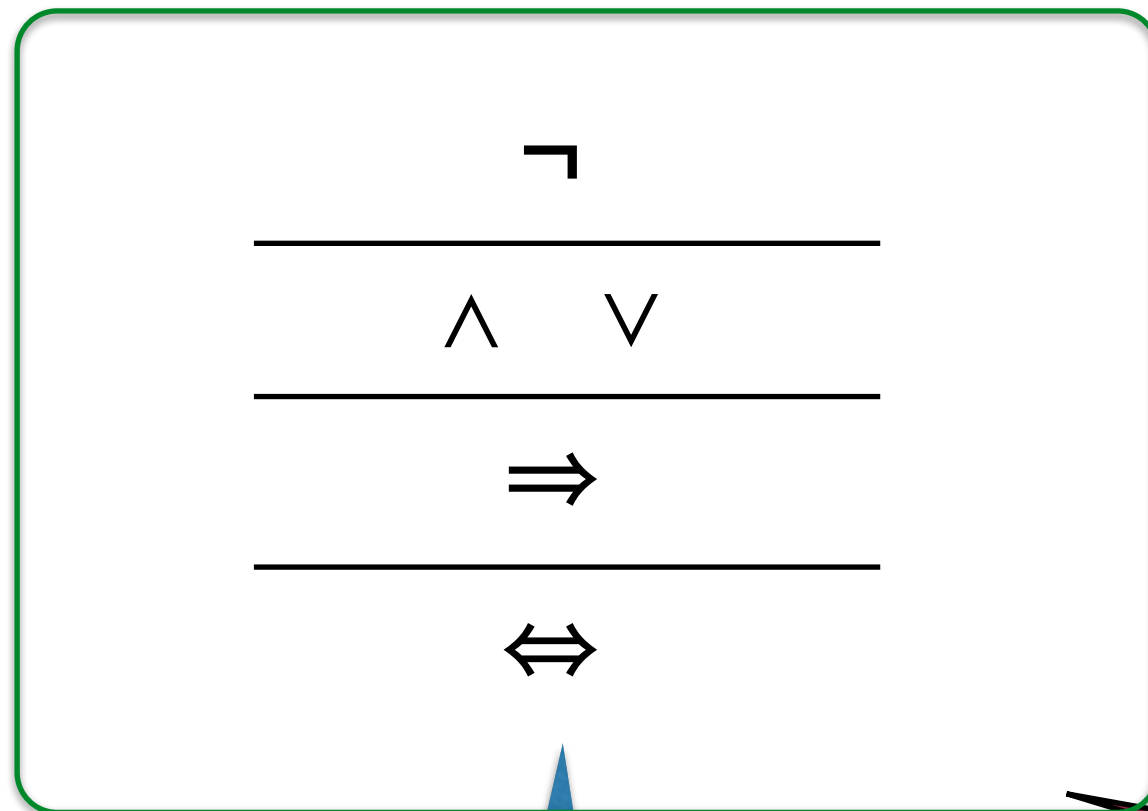
...and their structure



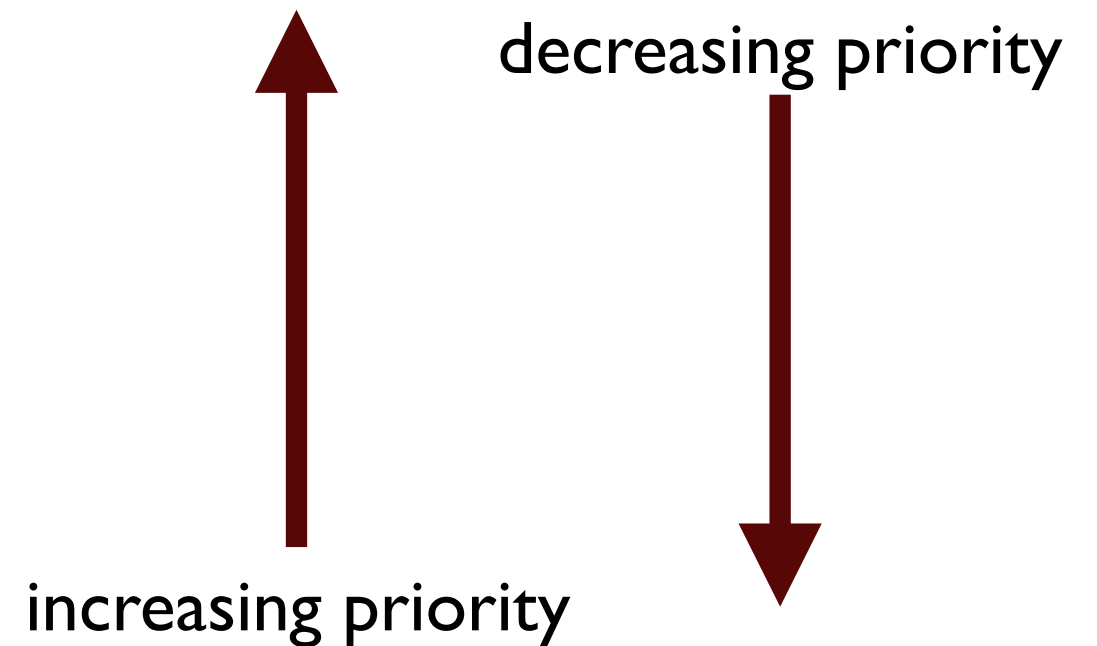
the tree of
 $((a \wedge b) \Rightarrow (\neg c))$

tree representation
(no need of
parenthesis)

Dropping parenthesis



priority schema
(top binds the most)



Example: $((a \wedge b) \Rightarrow (\neg c))$
becomes
 $a \wedge b \Rightarrow \neg c$

Truth tables

Conjunction

P	Q	$P \wedge Q$
0	0	0
0	1	0
1	0	0
1	1	1

only true when both
P and Q are true

Truth tables

Disjunction

P	Q	$P \vee Q$
0	0	0
0	1	1
1	0	1
1	1	1

true when either P
or Q or both are
true

Truth tables

Negation

unary connective

P	$\neg P$
0	1
1	0

true when P
is false

Truth tables

Implication

needs more attention

P	Q	$P \Rightarrow Q$
0	0	1
0	1	1
1	0	0
1	1	1

only false when P is true and Q is false

Truth tables

Bi-implication

$$P \Leftrightarrow Q$$

$$\text{is } (P \Rightarrow Q) \wedge (Q \Rightarrow P)$$

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \Leftrightarrow Q$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	1	1	1

true when P and Q have the same truth value