

## Example Test Tasks

to be discussed on 2.2.2018

**Task 1.** (20 points) Define the following notions:

- A relation  $R$  is a partial order.
- A relation  $R$  is irreflexive.
- A function  $f$  is injective.
- $|A| \leq |B|$  for two sets  $A$  and  $B$ .

**Task 2.** (20 points) Let  $X$  be a nonempty set and let  $\mathcal{R}_X$  denote the set of all relations on  $X$ . (We have  $\mathcal{R}_X = \mathcal{P}(X \times X)$ .)

Consider the function  $f$  on  $\mathcal{R}_X$  defined by  $f(R) = R \cup \Delta_X \cup R^{-1}$ . Express in your own words what  $f$  computes. To start with, for  $X = \{1, 2\}$  compute  $f(\Delta_X)$  and  $f(\{(1, 1), (1, 2)\})$ . Show that  $f$  is not injective.

**Task 3.** (20 points) Let  $\Sigma = \{0, 1\}$  and consider the relation  $\beta$  on  $\Sigma^*$  defined by

$$(u, v) \in \beta \quad \text{iff} \quad u = v \text{ or } u \text{ and } v \text{ both end with a } b.$$

Prove that  $\beta$  is an equivalence relation and describe its equivalence classes.

**Task 4.** (20 points) Prove that  $|a^*b| = |(aa)^*b| = \aleph_0$  by constructing suitable bijections.

**Task 5.** (20 points) Construct a DFA for the language

$$L = (ab \cup b)^*(aab)^*$$