Example Test Tasks

to be discussed on 12.12.17

Task 1. (20 points) Prove that the following propositional formula is a tautology:

$$\neg A_1 \wedge \neg A_2 \wedge \neg A_3 \wedge (\neg A_1 \wedge \neg A_2 \Rightarrow B) \wedge (\neg A_2 \wedge \neg A_3 \Rightarrow C) \Rightarrow B \wedge C.$$

Task 2. (20 points) Let A and B be arbitrary sets. Prove that the following two statements are equivalent:

$$A \subseteq B \quad \text{and} \quad A \cup B = B$$

Task 3. (20 points) Let $X = \{2, 4, 6, 8, 10\}$. Consider the predicate formula

$$\forall x\,[x \in X : \exists n\,[n \in \mathbb{N} : x = 2n]]\,.$$

(a) State the property in natural language.

(b) Is the property true?

Task 4. (20 points) Prove that

$$\forall x[P : R] \overset{val}{\models} \forall x[P : \neg Q \Rightarrow R] \wedge \neg \exists x[P : \neg R \wedge Q].$$

Task 5. (20 points) Is the following formula a tautology? Prove your answer.

$$(\forall x[P(x)] \wedge \forall x[R(x)]) \Rightarrow (\exists x[P(x) \Rightarrow Q(x)] \Rightarrow \exists x[Q(x)]).$$

In case you are bored, here is a little logic puzzle for you:

Suppose there are twin brothers; one which always tells the truth and one which always lies. What single yes/no question could you ask to either brother to figure out which one is which?

If you feel like, write the answer in your exam too.