Closure under regular operations

Theorem CI

The class of regular languages is closed under union

Theorem C2

The class of regular languages is closed under complement

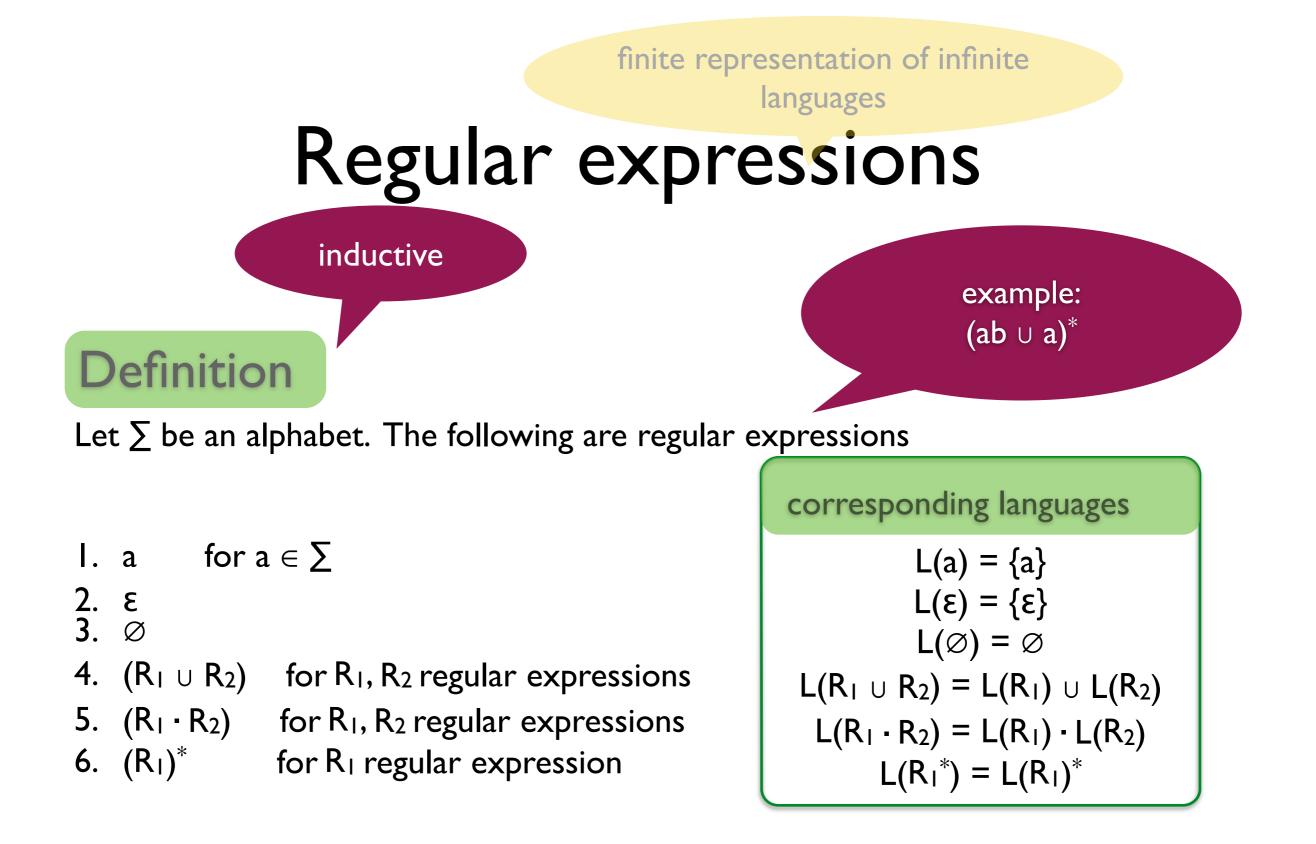
Theorem C3

The class of regular languages is closed under concatenation

Now we can prove these too

Theorem C4

The class of regular languages is closed under Kleene star



Equivalence of regular expressions and regular languages

Theorem (Kleene)

A language is regular (i.e., recognised by a finite automaton) iff it is the language of a regular expression.

Proof \Leftarrow easy, as the constructions for

the closure properties, \Rightarrow not so easy, we'll skip it for now...

Nonregular languages

every long enough word of a regular language can be pumped

Theorem (Pumping Lemma)

If L is a regular language, then there is a number $p \in \mathbb{N}$ (the pumping length) such that for any $w \in L$ with $|w| \ge p$, there exist x, y, $z \in \sum^*$ such that w = xyz and 1. $xy^iz \in L$, for all $i \in \mathbb{N}$

- 2. |y| > 0
- 3. |xy| ≤p

Proof easy, using the pigeonhole principle

Example "corollary"

L= { $0^n 1^n \mid n \in \mathbb{N}$ } is nonregular.