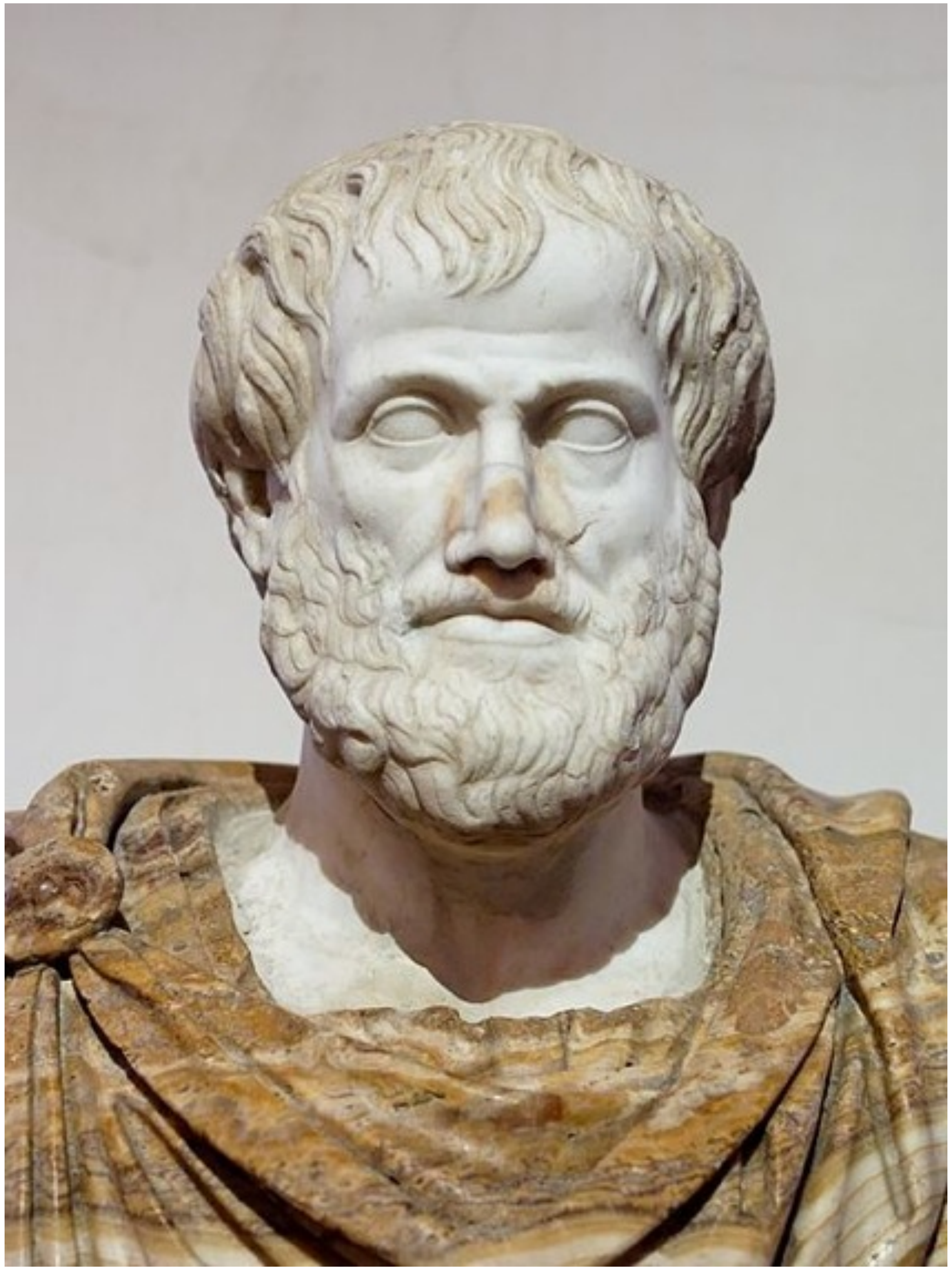
Logic

In the beginning



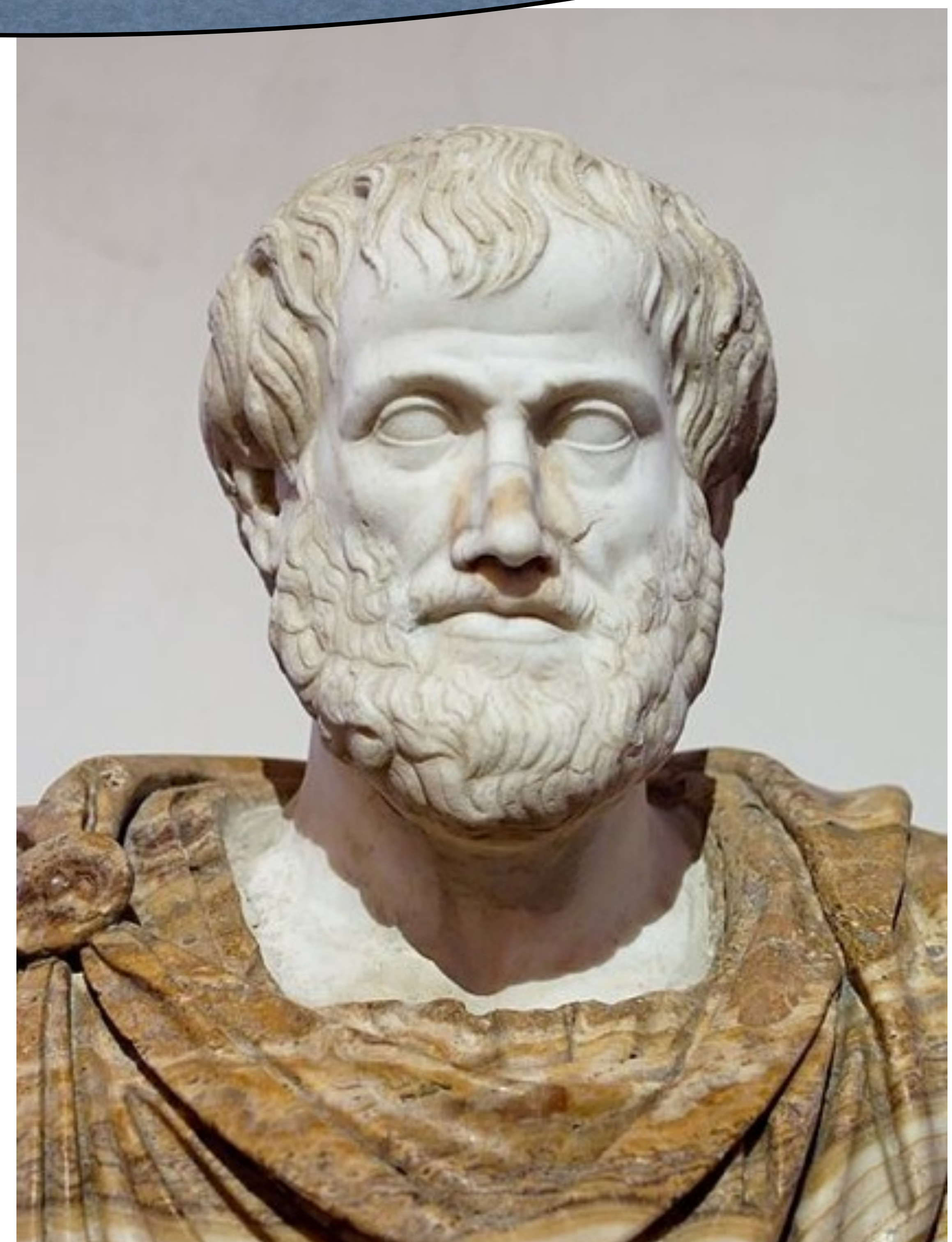
Aristotle +/- 350 B.C.

Organon

19 syllogisms

Logic = study of correct reasoning

In the beginning



Aristotle +/- 350 B.C.

Organon

19 syllogisms

Barbara syllogism

All K's are L's
All L's are M's

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only later called so, in the Middle Ages

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one can always conclude the conclusion

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one can **always** conclude the conclusion

independent of what the parameters K,L,M are

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from the two premises

one can always conclude the conclusion

independent of what the parameters K,L,M are

Logic (Logos, Greek for word, understanding, reason) deals with general reasoning laws in the form of formulas with parameters.

Propositions

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Connectives

- $\wedge$ for "and"
- $\vee$ for "or"
- $\neg$ for "not"
- $\Rightarrow$ for "if .. then" or "implies"
- $\Leftrightarrow$ for "if and only if"

Abstract propositions

Abstract propositions

Definition

Basis	Propositional variables are abstract propositions.
Step (Case 1)	If P is an abstract proposition, then so is $(\neg P)$.
Step (Case 2)	If P and Q are abstract propositions, then so are $(P \wedge Q)$, $(P \vee Q)$, $(P \Rightarrow Q)$, and $(P \Leftrightarrow Q)$.

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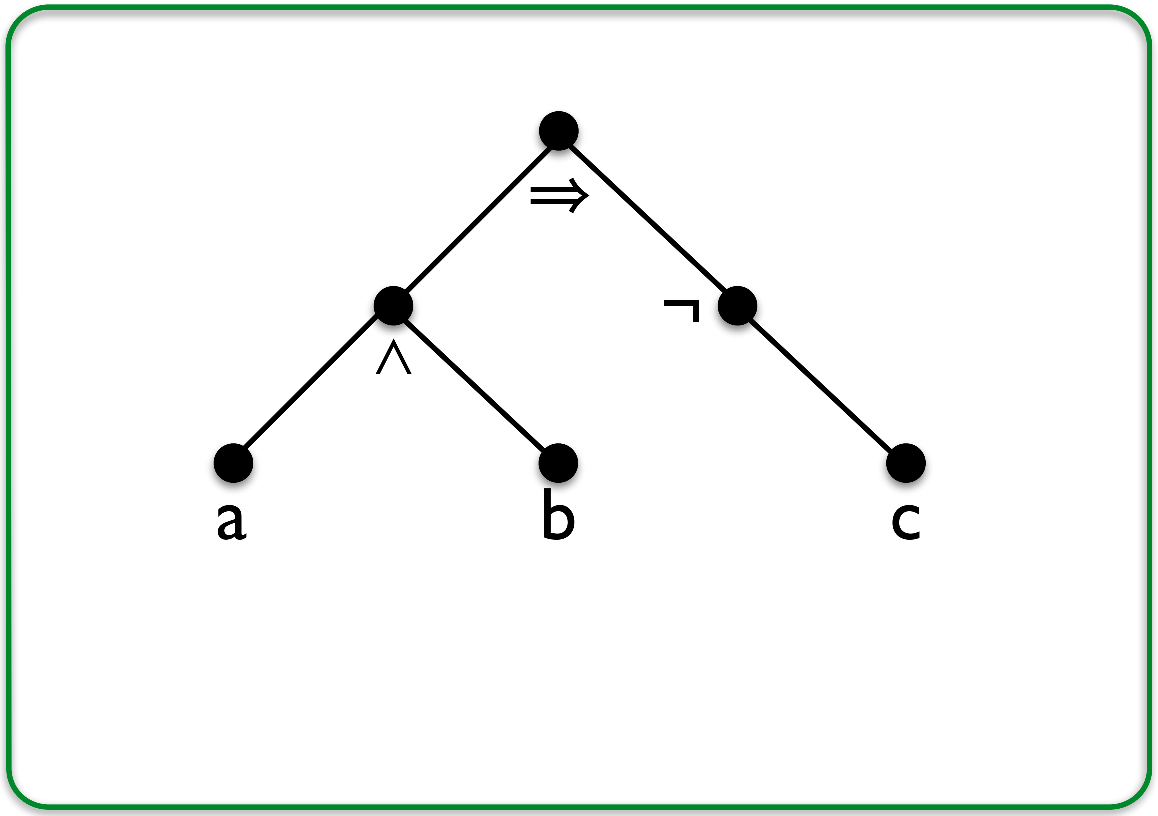
a recursive/inductive definition

…and their structure



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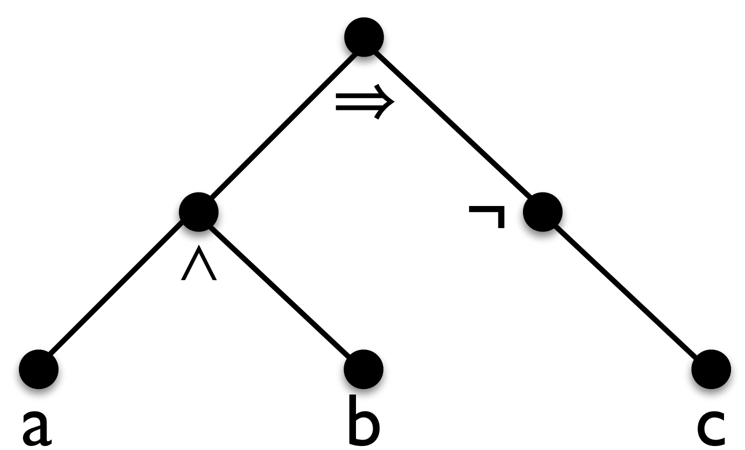
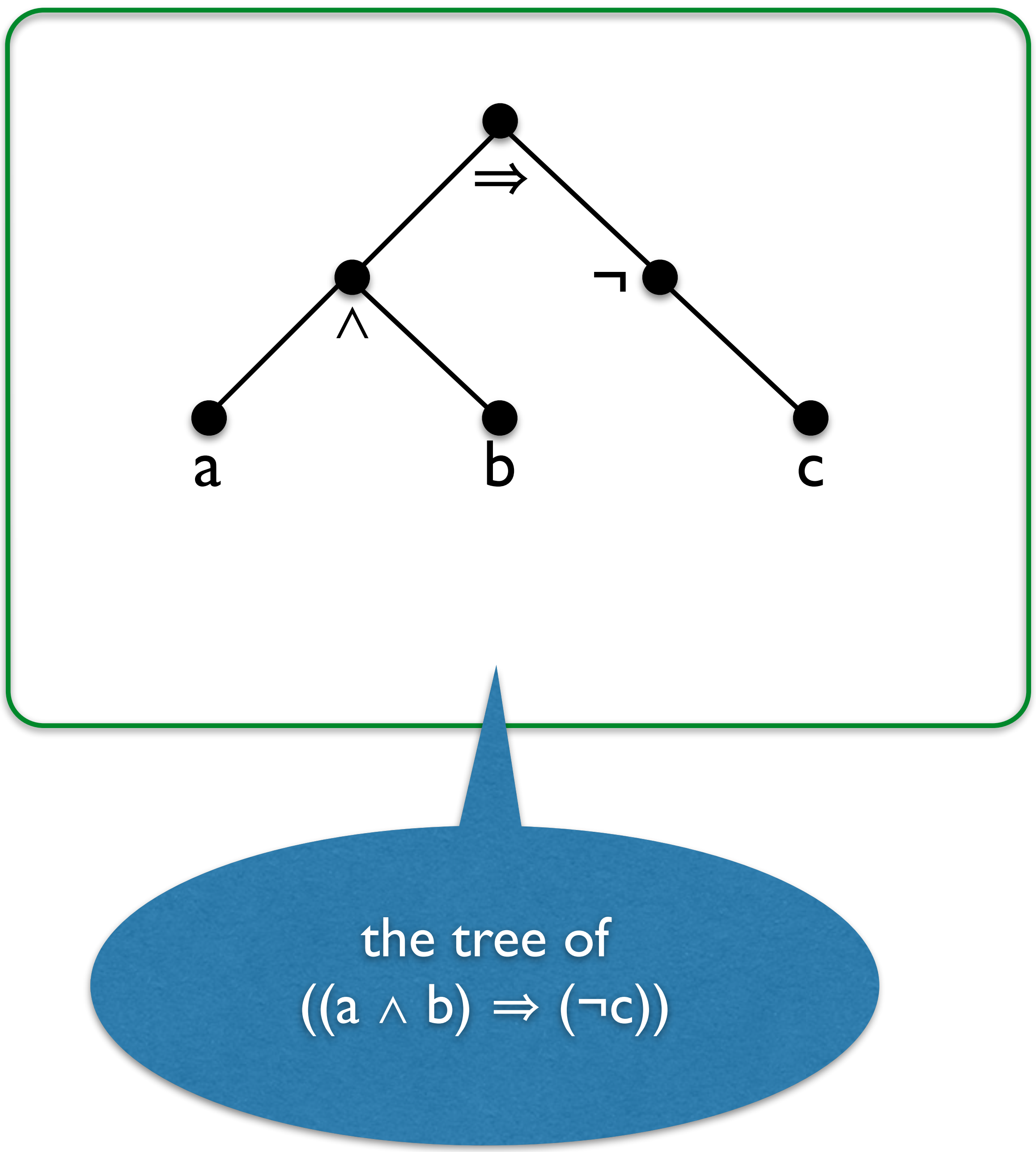
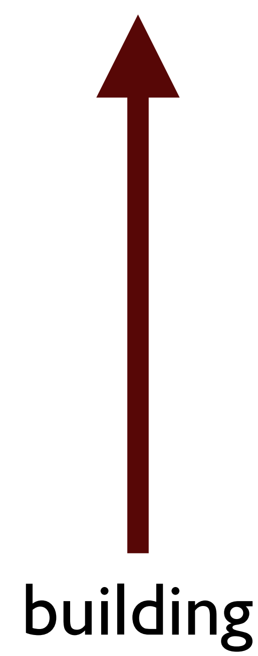


the tree of
$((a \wedge b) \Rightarrow (\neg c))$

…and their structure





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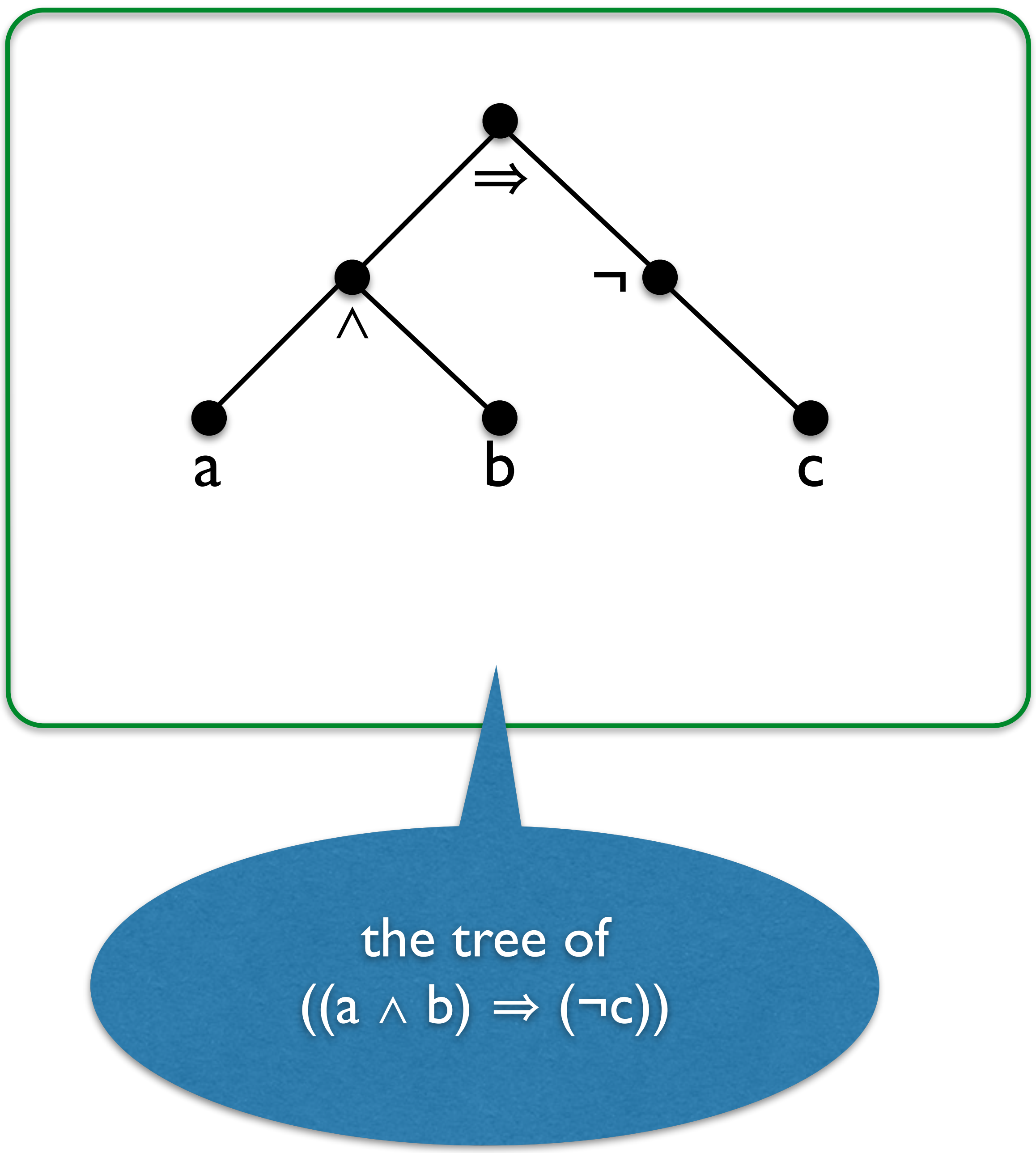

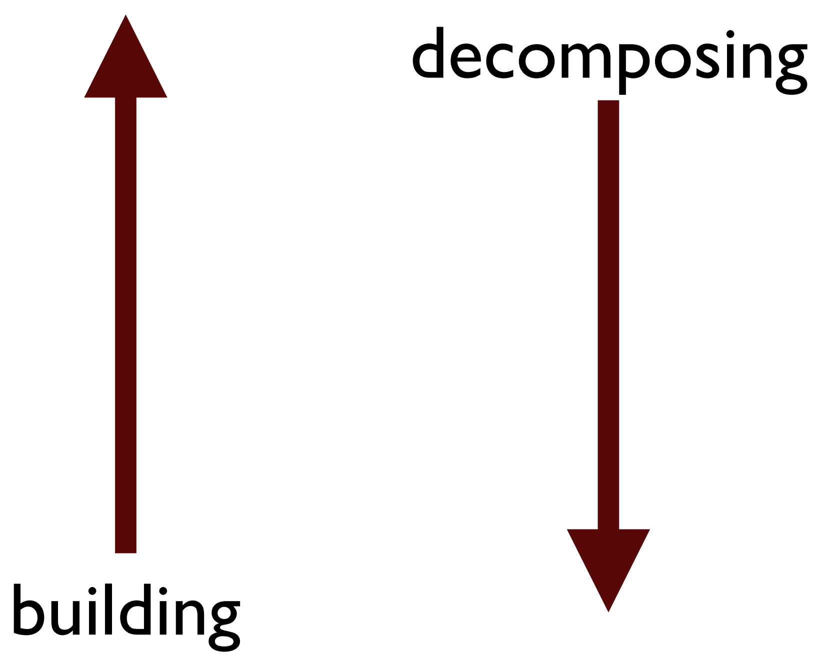

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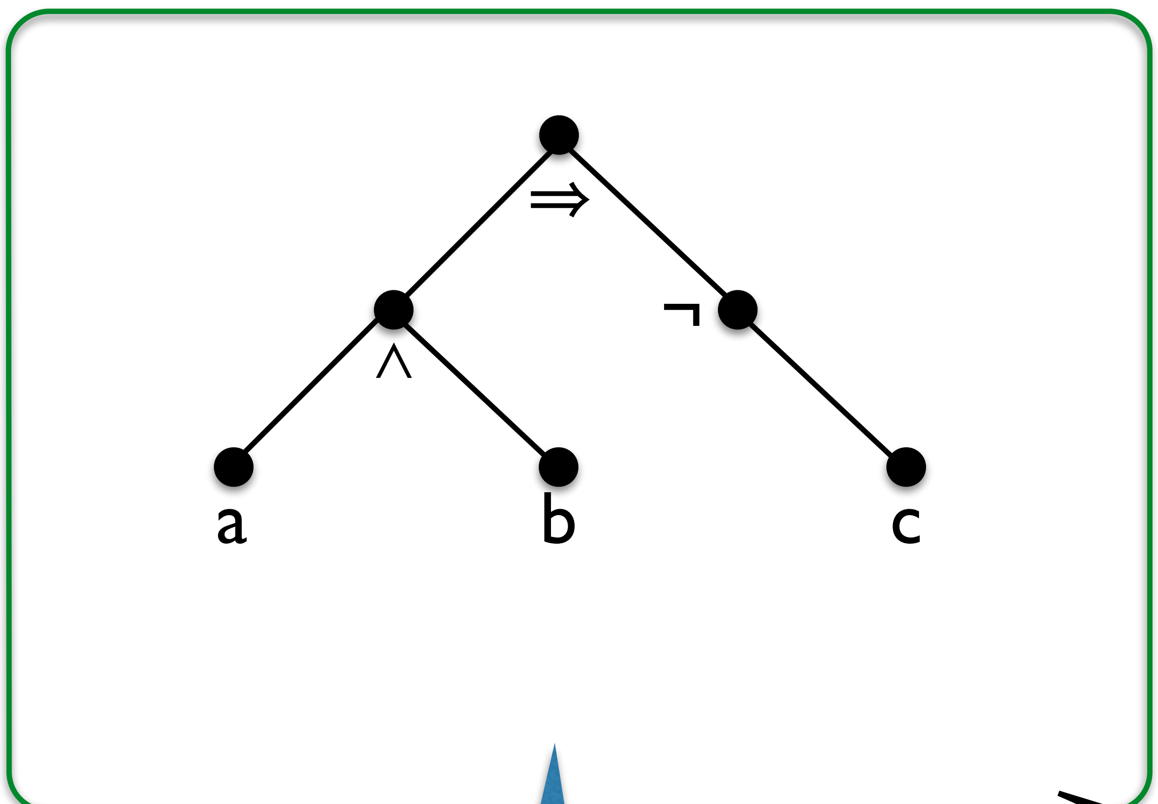

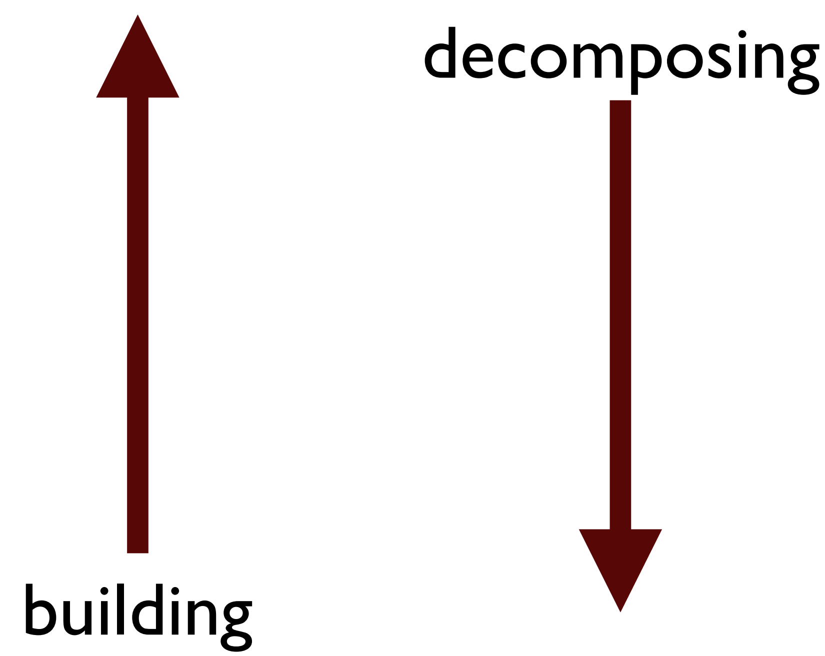

the tree of
$((a \wedge b) \Rightarrow (\neg c))$

tree representation
(no need of
parenthesis)

Dropping parenthesis

$\neg$
$\wedge \quad \vee$
$\Rightarrow$
$\Leftrightarrow$

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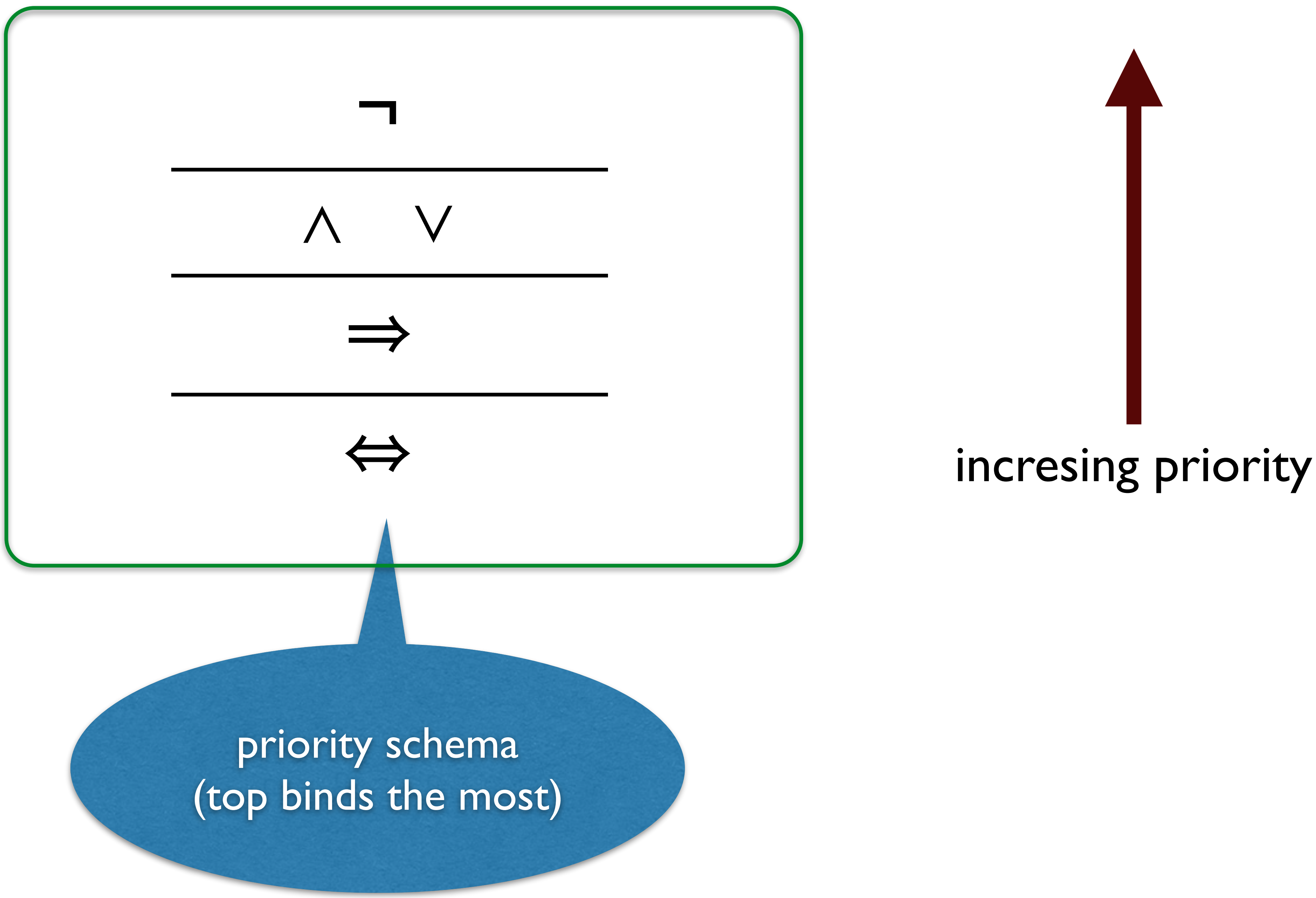
priority schema
(top binds the most)

Dropping parenthesis



Dropping parenthesis

$\neg$

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priority schema
(top binds the most)

decreasing priority

incresing priority

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Example: $((a \wedge b) \Rightarrow (\neg c))$

becomes

$a \wedge b \Rightarrow \neg c$

Truth tables

Conjunction

P	Q	P∧Q
0	0	0
0	1	0
1	0	0
1	1	1

Truth tables

Conjunction

P	Q	$P \wedge Q$
0	0	0
0	1	0
1	0	0
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Truth tables

Conjunction

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only true when both
P and Q are true

Truth tables

Disjunction

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Truth tables

Negation

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unary connective

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Truth tables

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Truth tables

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true when P
is false

Truth tables

Implication

Truth tables

Implication

needs more attention

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P	Q	$P \Rightarrow Q$
0	0	1
0	1	1
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needs more attention

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only false when P is true and Q is false

Truth tables

Bi-implication

Truth tables

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$P \Leftrightarrow Q$

is $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

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true when P and Q have the same truth value

Truth-functions

Def. A truth-function or Boolean function is a function

$$f\colon \{0,1\}^n \longrightarrow \{0,1\}$$

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Notation in the book…

$$\begin{array}{l} \quad a, b \\ \left\{ \begin{array}{l} (0,0) \longmapsto 0 \\ (0,1) \longmapsto 1 \\ (1,0) \longmapsto 0 \\ (1,1) \longmapsto 1 \end{array} \right. \end{array}$$

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$P(a,b)$: $(a \wedge b) \vee b$

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$a_1, .. a_n$ are the variables in P (and more) ordered in a sequence

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P(a,b): $(a \wedge b) \vee b$

Truth-functions

$a_1, .. a_n$ are the variables in P (and more) ordered in a sequence

Property: Every abstract proposition $P(a_1,..,a_n)$ with ordered and specified variables induces a truth-function.

Note:

The sequence of specified variables matters!

$P(a,b,c)$: $(a \wedge b) \vee b$

induces

$$
\begin{array}{l}
a, b, c \\
(0,0,0) \longmapsto 0 \\
(0,0,1) \longmapsto 0 \\
(0,1,0) \longmapsto 1 \\
(0,1,1) \longmapsto 1 \\
(1,0,0) \longmapsto 0 \\
(1,0,1) \longmapsto 0 \\
(1,1,0) \longmapsto 1 \\
(1,1,1) \longmapsto 1
\end{array}
$$

Equivalence of propositions

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on any sequence containing their common variables

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Example

Are the following equivalent? $b \wedge \neg b$ and $c \wedge \neg c$

b	c	$\neg b$	$\neg c$	$b \wedge \neg b$	$c \wedge \neg c$
0	0				
0	1				
1	0				
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0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

Their truth values are the same, so they are equivalent

$$b \wedge \neg b \stackrel{val}{=} c \wedge \neg c$$