# Properties of sets

| | |
|---|---|
| 1. | $\varnothing \subseteq X$ |
| 2. | If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$ |
| 3. | $X \cap Y \subseteq X$, $X \cap Y \subseteq Y$ |
| 4. | $X \subseteq X \cup Y$, $Y \subseteq X \cup Y$ |
| 5. | If X |
| 6. | If X |
| 7. | $X \cap Y = X$ iff $X \subseteq Y$ |
| 8. | $X \cap X = X$ (idempotence) |
| 9. | $X \cup X = X$ (idempotence) |
| 10. | $X \cap \varnothing = \varnothing$ |

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| 11. | $X \cup \emptyset = X$ |
| 12. | $X \cap Y = Y \cap X$ (commutativity) |
| 13. | $X \cup Y = Y \cup X$ (commutativity) |
| 14. | $X \cap (Y \cap Z) = (X \cap Y) \cap Z$ (associativity) |
| 15. | $X \cup (Y \cup Z) = (X \cup Y) \cup Z$ (associativity) |
| 16. | $X \cap (X \cup Y) = X$ (absorption) |
| 17. | $X \cup (X \cap Y) = X$ (absorption) |
| 18. | $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$ (distributivity) |
| 19. | $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$ (distributivity) |
| 20. | $X \setminus Y \subseteq X$ |

# Properties of sets

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| 21. | $(X \setminus Y) \cap Y = \varnothing$ |
| 22. | $X \cup Y = X \cup (Y \setminus X)$ |
| 23. | $X \setminus X = \varnothing$ |
| 24. | $X \setminus \varnothing = X$ |
| 25. | $\varnothing \setminus X = \varnothing$ |
| 26. | If $X \subseteq Y$, then $X \setminus Y = \varnothing$ |
| 27. | (X |
| 28. | (X |
| 29. | (X |
| 30. | $X \times \varnothing = \varnothing$ |
| 31. | $\varnothing \times X = \varnothing$ |
| 32. | If $X \subseteq Y$, then $\mathcal{P}(X) \subseteq \mathcal{P}(Y)$ |

# Product of multiple sets

Direct product (Kartesisches Produkt)

$$A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$$

ordered pairs

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In general, for sets $A_1, A_2, \ldots, A_n$ with $n \geq 1$,

$$A_1 \times A_2 \times \ldots \times A_n = \prod_{1 \leq i \leq n} A_i = \{(x_1,x_2,\ldots,x_n) \mid x_i \in A_i \text{ for } 1 \leq i \leq n\}$$

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if $A_i = A$ for all i, then the product is denoted $A^n$

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Let A be a set, an aphabet

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# Relations

Def. If A and B are sets, then any subset $R \subseteq A \times B$ is a (binary) relation between A and B

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some relations are special

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similarly, unary relation (subset), n-ary relation...

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some relations are special

# Special relations

A relation $R \subseteq A \times A$ is:

| | | |
|---|---|---|
| reflexive | iff | for all $a \in A$, $(a,a) \in R$ |
| symmetric | iff | for all $a,b \in A$, if $(a,b) \in R$, then $(b,a) \in R$ |
| transitive | iff | for all $a,b,c \in A$, if $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$ |
| irreflexive | iff | for all $a \in A$, $(a,a) \notin R$ |
| antisymmetric | iff | for all $a,b \in A$, if $(a,b) \in R$ and $(b,a) \in R$ then $a = b$ |
| asymmetric | iff | for all $a,b \in A$, if $(a,b) \in R$, then $(b,a) \notin R$ |
| total | iff | for all $a,b \in A$, $(a,b) \in R$ or $(b,a) \in R$ |

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(infix) notation $aRb$ for $(a,b) \in R$