Proofs with 3-introduction and 3elimination are unnecessarily long and cumbersome...

There are alternatives!

Proving an existential quantification

To prove

∃x[x ∈ ℤ : x³ - 2x - 8 ≥0]

Proof

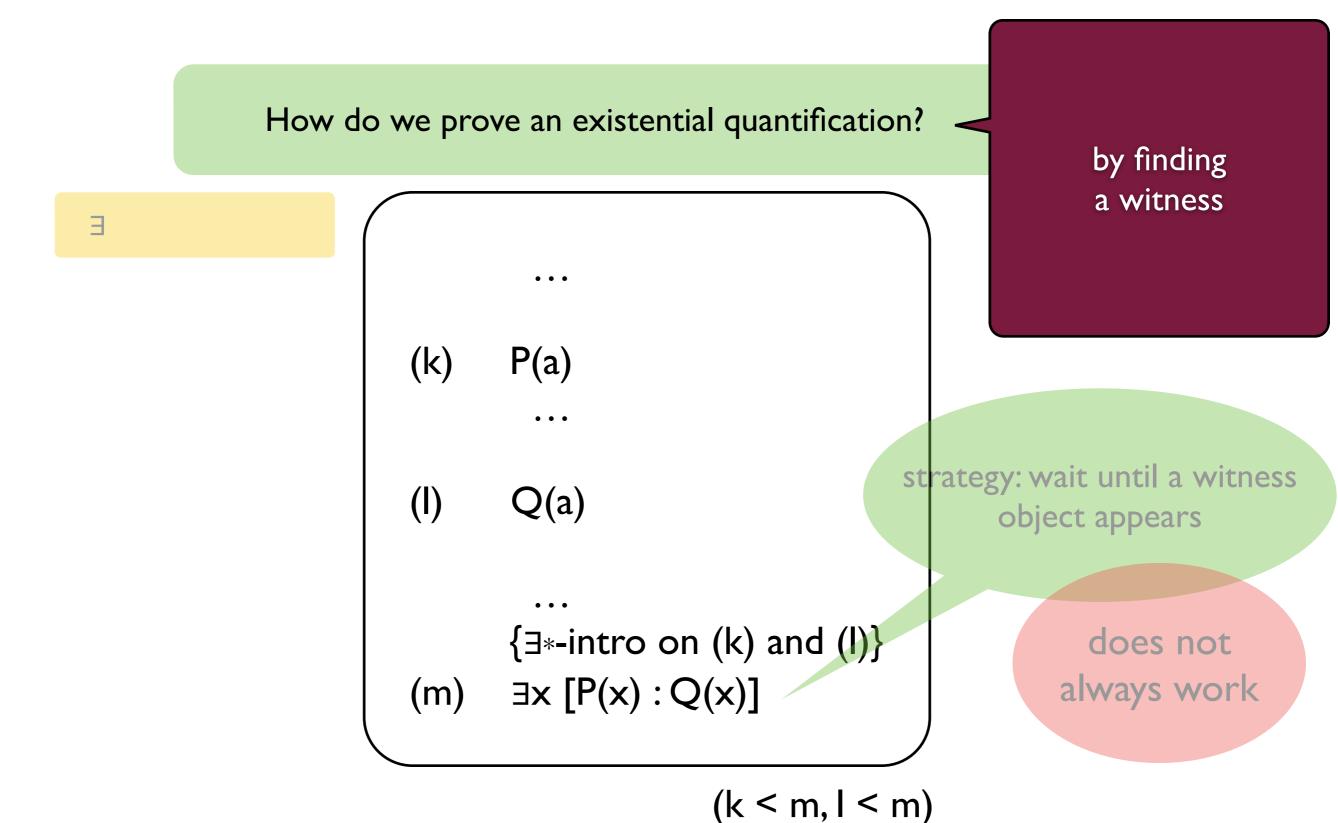
It suffices to find a witness, i.e., an $x \in \mathbb{Z}$ satisfying $x^3 - 2x - 8 \ge 0$.

x = 3 is a witness, since $3 \in \mathbb{Z}$ and $3^3 - 2 \cdot 3 - 8 = 13 \ge 0$

Conclusion: $\exists x[x \in \mathbb{Z} : x^3 - 2x - 8 \ge 0].$

also x = 5 is a witness...

Alternative 3 introduction



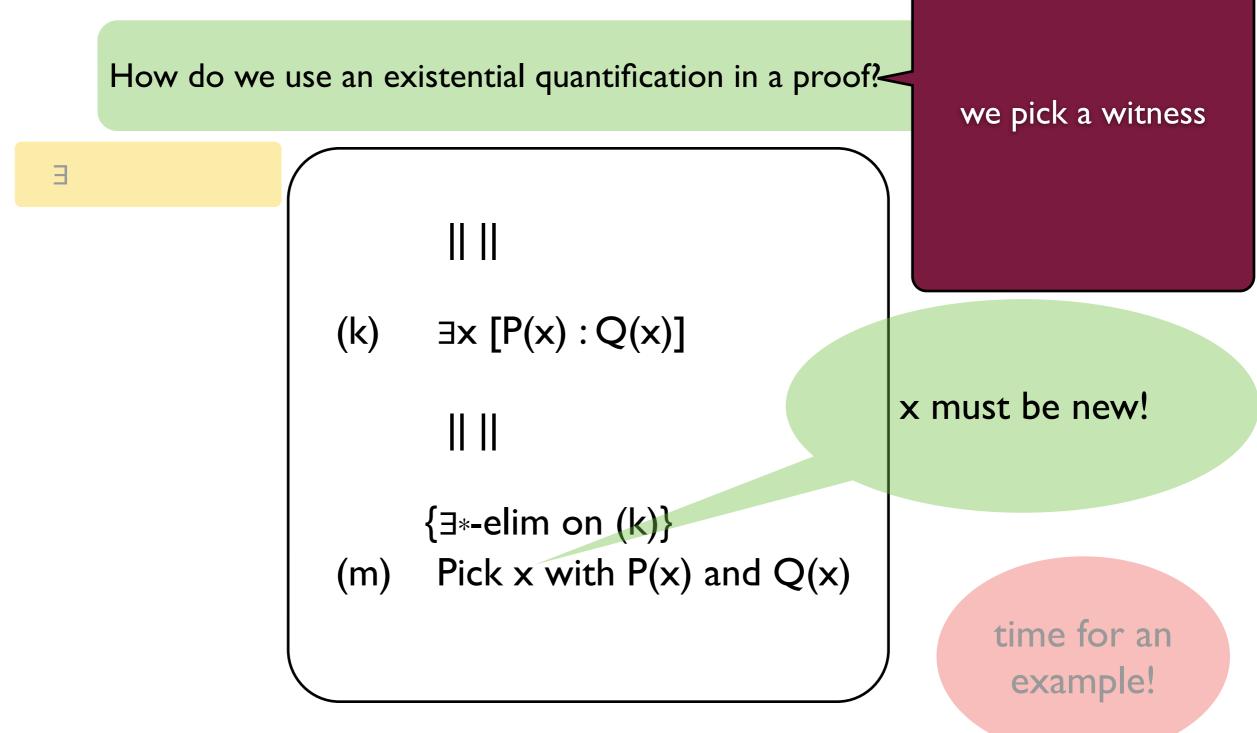
Using an existential quantification

We know

$$\exists x [x \in \mathbb{R} : a - x < 0 < b - x]$$

We can declare an
$$x \in \mathbb{Z}$$
 (a witness) such that
 $a - x < 0 < b - x$
and use it further in the proof. For example:
From $a - x < 0$, we get $a < x$.
From $b - x > 0$, we get $x < b$.
Hence, $a < b$.

Alternative 3 elimination



(k < m)