Finite Automata

Alphabets and Languages

Recall

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Σ - alphabet (finite set)

$\Sigma^n = \{a_1 a_2 .. a_n \mid a_i \in \Sigma\}$ is the set of words of length n

$\Sigma^* = \{w \mid \exists n \in \mathbb{N}. \exists\, a_1, a_2, .. , a_n \in \Sigma. w = a_1 a_2 .. a_n\}$ is the set of all words over Σ

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$\Sigma^0 = \{\varepsilon\}$ contains only the empty word

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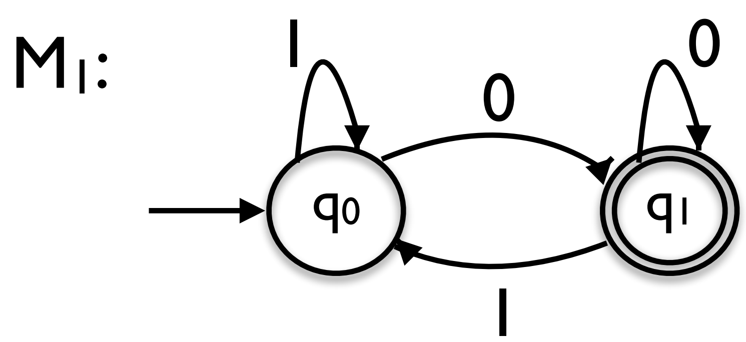
A language L over Σ is a subset $L \subseteq \Sigma^*$

Deterministic Automata (DFA)

Informal example

$\Sigma = \{0,1\}$



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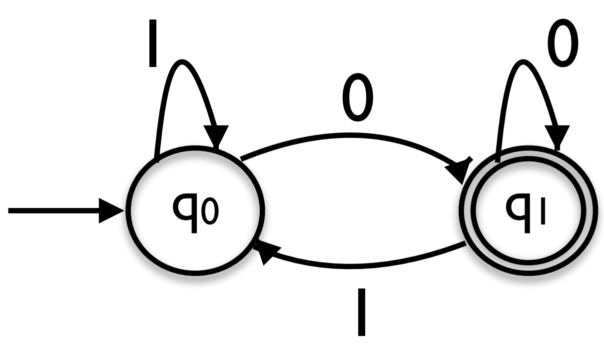

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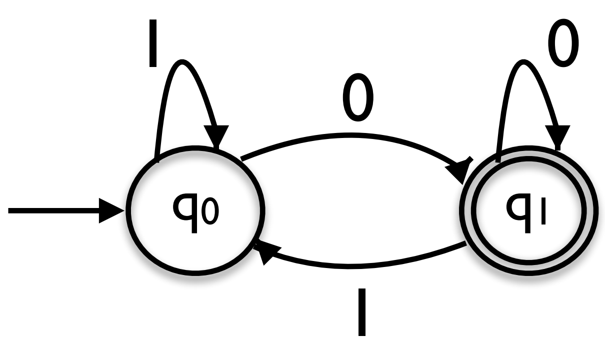

q_0, q_1 are states

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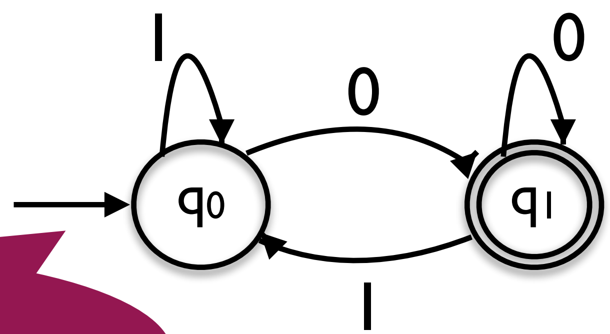

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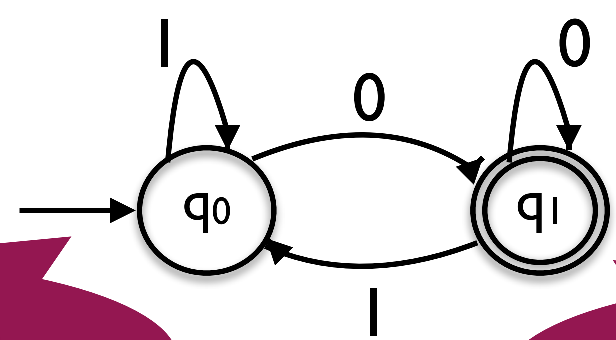

q_0, q_1 are states

q_0 is initial

q_1 is final

Deterministic Automata (DFA)

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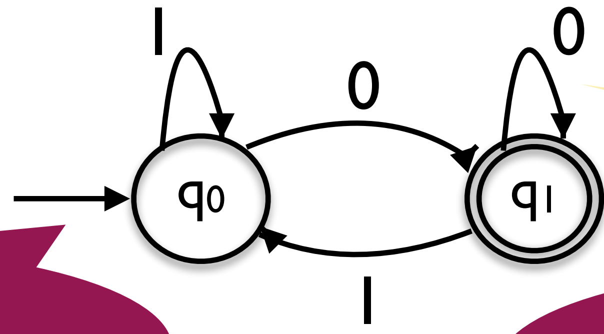

q_0, q_1 are states

transitions, labelled by alphabet symbols

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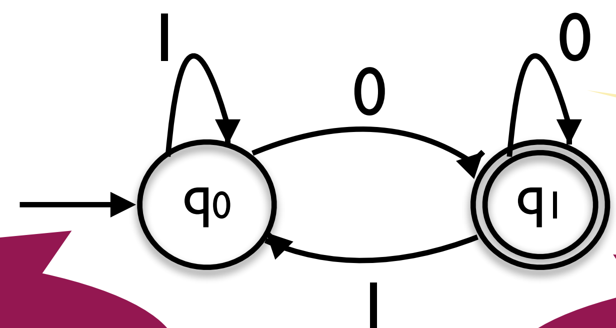

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Accepts the language $L(M_1) = \{w \in \Sigma^* \mid w \text{ ends with a } 0\} = \Sigma^* 0$

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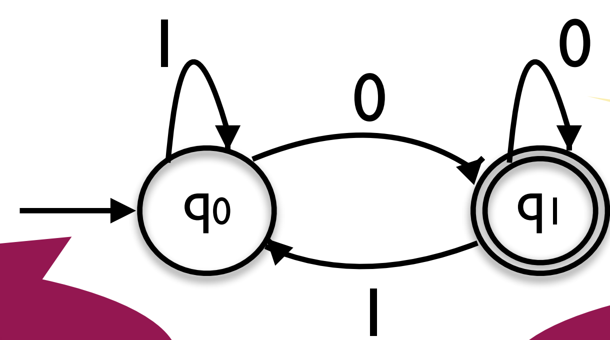

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regular language

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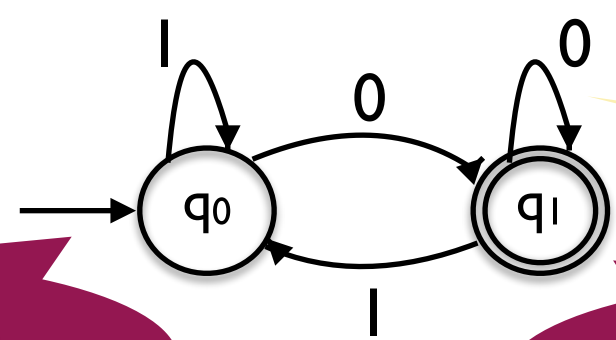

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regular language

regular expression

DFA

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DFA

The extended transition function

DFA

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The language recognised / accepted by a deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ is

$$L(M) = \{w \in \Sigma^* \mid \delta^*(q_0, w) \in F\}$$

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$L(M_1) = \{w0 | w \in \{0,1\}^*\}$

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Regular languages and operations

Definition

Let Σ be an alphabet. A language L over Σ ($L \subseteq \Sigma^*$) is regular iff it is recognised by a DFA.

Regular languages and operations

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$\varepsilon \in L^*$ always

Closure under regular operations

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Theorem C1

The class of regular languages is closed under union

Closure under regular operations

also under intersection

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The class of regular languages is closed under union

Theorem C2

The class of regular languages is closed under complement

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The class of regular languages is closed under union

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Theorem C3

The class of regular languages is closed under concatenation

Closure under regular operations

also under intersection

Theorem C1

The class of regular languages is closed under union

Theorem C2

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Theorem C4

The class of regular languages is closed under Kleene star

Closure under regular operations

also under intersection

Theorem C1

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We can already prove these!

Theorem C2

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Theorem C3

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Theorem C4

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also under intersection

Theorem C1

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Theorem C2

The class of regular languages is closed under complement

Theorem C3

The class of regular languages is closed under concatenation

But not yet these two…

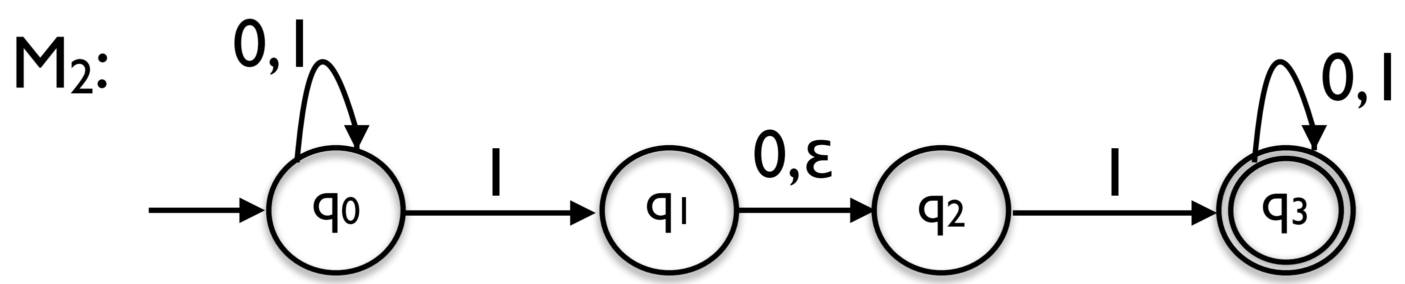
Theorem C4

The class of regular languages is closed under Kleene star

Nondeterministic Automata (NFA)

Informal example

$\Sigma = \{0,1\}$



Nondeterministic Automata (NFA)

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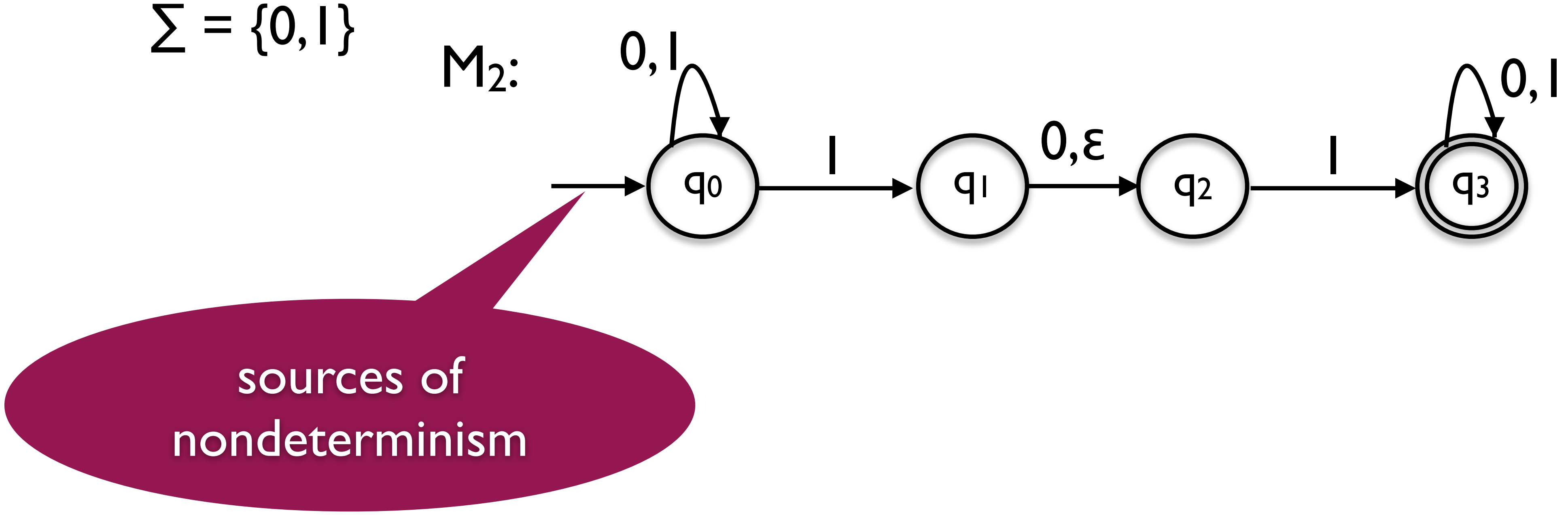

Nondeterministic Automata (NFA)

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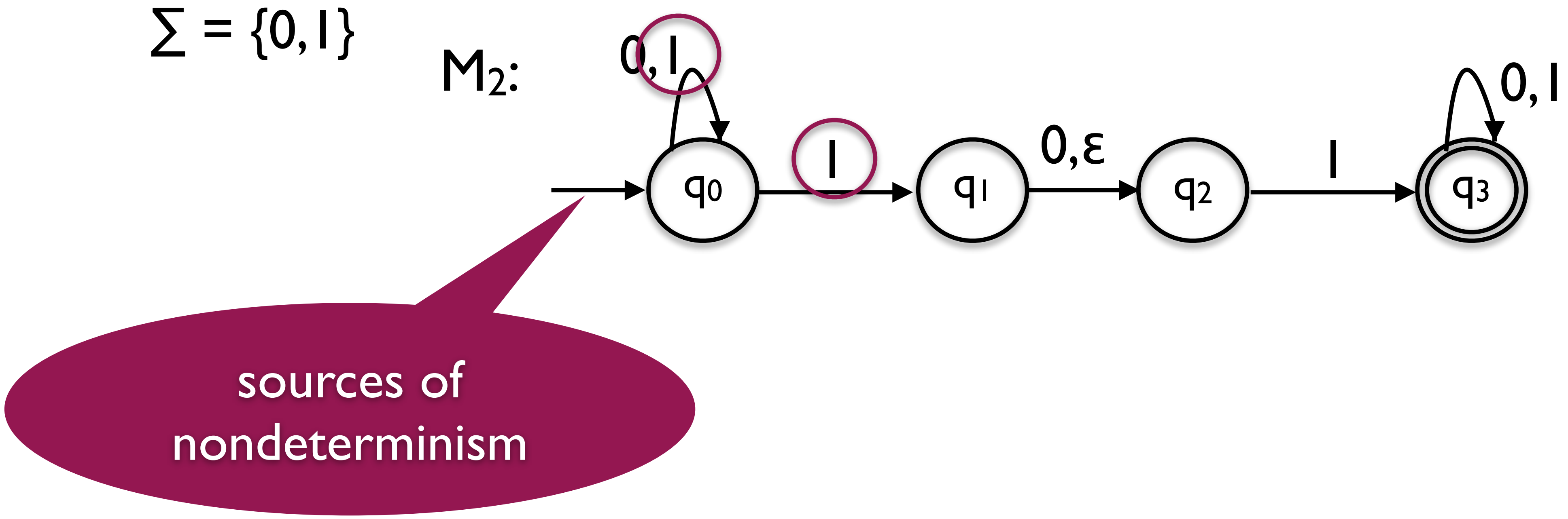

Nondeterministic Automata (NFA)

Informal example

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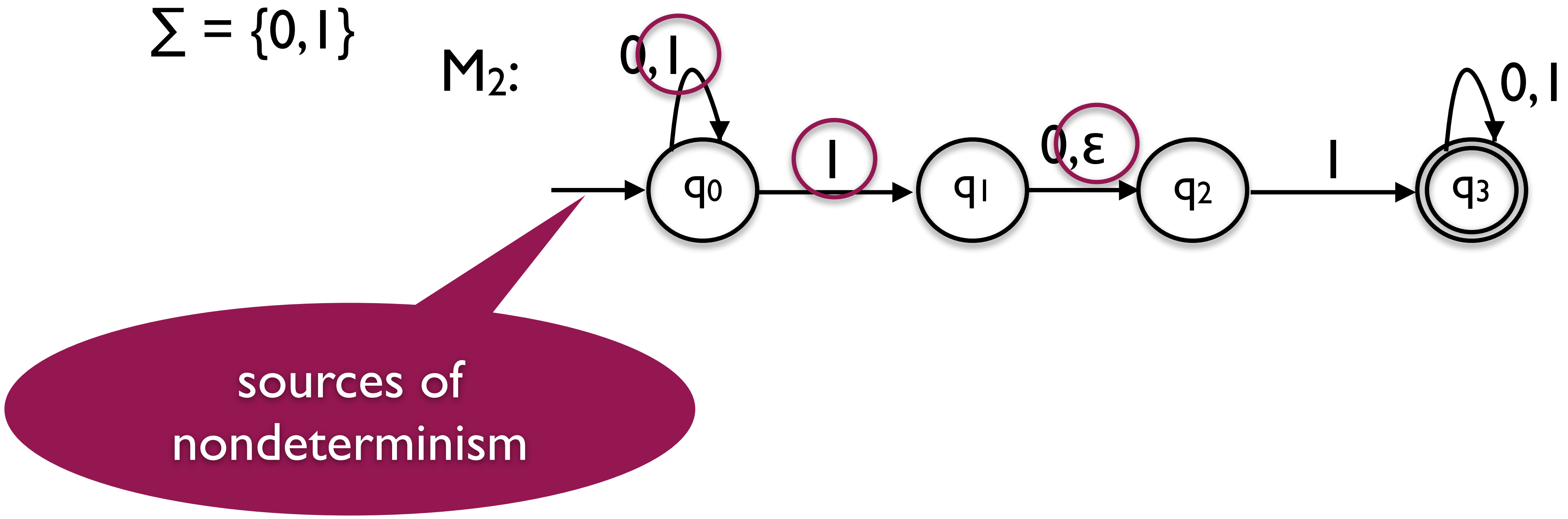

Nondeterministic Automata (NFA)

no 1 transition

Informal example

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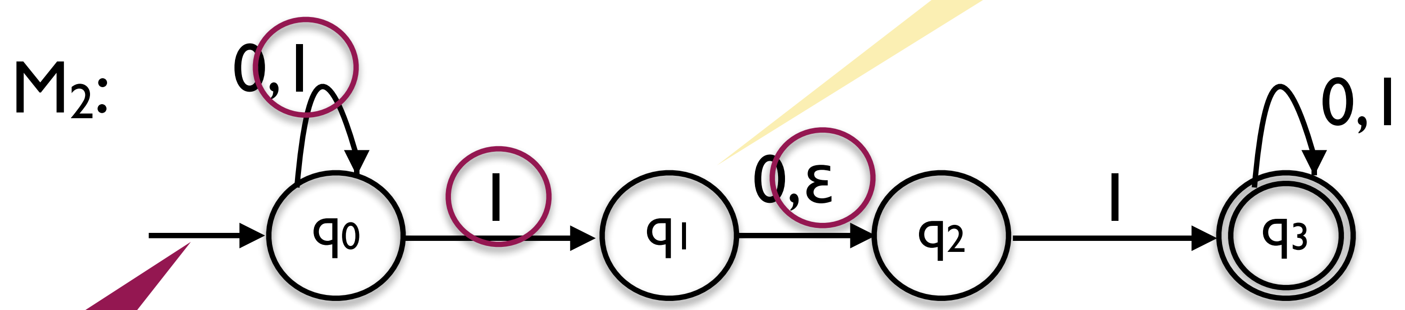

sources of nondeterminism

Nondeterministic Automata (NFA)

Informal example

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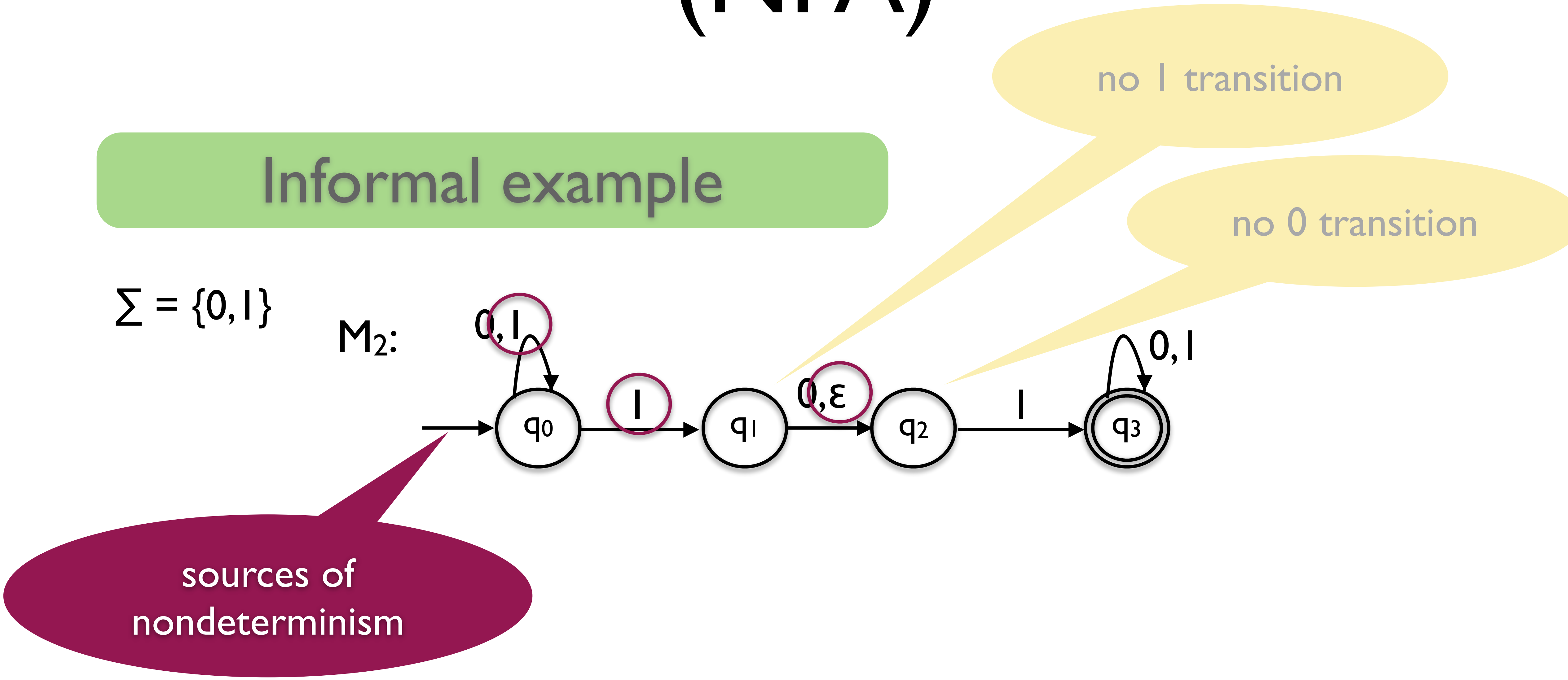

Nondeterministic Automata (NFA)

no 1 transition

Informal example

no 0 transition

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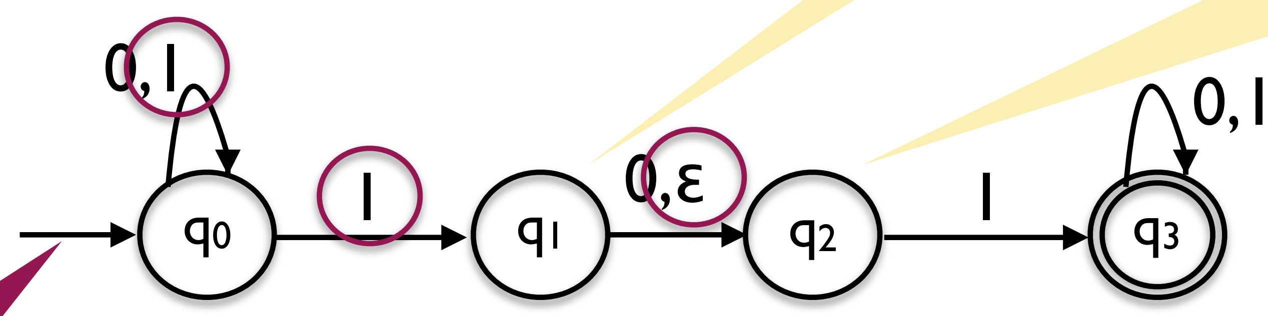
M_2:



sources of nondeterminism

Accepts a word iff there exists an accepting run

NFA

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$\delta(q_0, 0) = \{q_0\}$
$\delta(q_0, 1) = \{q_0, q_1\}$
$\delta(q_0, \varepsilon) = \varnothing$
…..

NFA

The extended transition function

NFA

The extended transition function

Given an NFA $M = (Q, \Sigma, \delta, q_0, F)$ we can extend $\delta: Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$ to

$\delta^*: Q \times \Sigma^* \longrightarrow \mathcal{P}(Q)$

inductively, by:

$\delta^*(q, \varepsilon) = E(q)$ and $\delta^*(q,wa) = E(\bigcup_{q' \in \delta^*(q,w)} \delta(q', a))$

NFA

$E(q) = \{q' \mid q' = q \vee \exists n \in \mathbb{N}^+ . \exists q_0, .., q_n \in Q . q_0 = q, q_n = q', q_{i+1} \in \delta(q_i, \varepsilon), \text{ for } i = 0, .., n\text{-}1\}$

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ε-closure of q, all states reachable by ε-transitions from q

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$E(X) = \bigcup_{x \in X} E(x)$

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$\delta^*(q, \varepsilon) = E(q)$ and $\delta^*(q, wa) = E(\bigcup_{q' \in \delta^*(q,w)} \delta(q', a))$

$E(X) = \bigcup_{x \in X} E(x)$

In M_2, $\delta^*(q_0, 0110) = \{q_0, q_2, q_3\}$

ε-closure of q, all states reachable by ε-transitions from q

NFA

$E(q) = \{q' \mid q' = q \vee \exists n \in \mathbb{N}^+ . \exists q_0, .., q_n \in Q . q_0 = q, q_n = q', q_{i+1} \in \delta(q_i, \varepsilon), \text{ for } i = 0, .., n-1\}$

The extended transition function

Given an NFA $M = (Q, \Sigma, \delta, q_0, F)$ we can extend $\delta : Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$ to

$\delta^* : Q \times \Sigma^* \longrightarrow \mathcal{P}(Q)$

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Definition

The language recognised / accepted by a nondeterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ is

$$L(M) = \{w \in \Sigma^* \mid \delta^*(q_0, w) \cap F \neq \emptyset\}$$

ε-closure of q, all states reachable by ε-transitions from q

NFA

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Definition

The language recognised / accepted by a ... automaton M = $(Q, \Sigma, \delta, q_0, F)$ is

$L(M_2) = \{u101w \mid u,w \in \{0,1\}^*\} \cup \{u11w \mid u,w \in \{0,1\}^*\}$

$$L(M) = \{w \in \Sigma^* \mid \delta^*(q_0, w) \cap F \neq \varnothing\}$$