LOGIC and SETS

Formale Systeme

3VO + 2PS

Lecturer: Dr. Ana Sokolova

Instructions: Dr. Ana Sokolova + Prof. Robert Elsässer

http://cs.uni-salzburg.at/~anas/Ana_Sokolova/teaching/FormaleSysteme2013/

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The Rules of the Game

- Lectures Thursday 1:15 pm - 4 pm.
- Instructions
  Group 1, Wednesday 1:15 pm - 3 pm (AS)
  Group 2, Thursday 10:15 am - 12am (RE)
- Tutors Cornelia Mayer and Markus Reiter
  Tuesday 12am-1pm
- Books

  Logical Reasoning: A First Course by R. Nederpelt and F. Kamaraddine

  Modellierung: Grundlagen und formale Methoden by U. Kastens and H. Kleine Büning

  Introduction to Automata Theory, Languages, and Computation by J. E. Hopcroft, R. Motwani and J.D. Ullman

The Rules... Instructions (PS)

Starting in 2 weeks!

- Instruction exercises on the web http://cs.uni-salzburg.at/~anas/Ana_Sokolova/teaching/FormaleSystemeProseminar2013/ on Thursday afternoons
- To be solved by you (in groups of at most 3 students) and handed in as homework to the instruction lecturer before Wednesday 11am
- In class we will present a sample solution and the students will be asked to present solutions/discuss the exercises

The Rules... Instructions (PS)

- One randomly chosen exercise will be graded each week
- The graded exercise will be returned to you in a week
- Grade based on
  (1) the grades of the corrected exercise and
  (2) activity in class (ability to present solutions)
- All information about the course / rules / exams / grading is / will be on the course webpage

The Rules... Exam (VO)

- Written exams
- Written exam in February, April, and July or two partial tests during the semester
- Grade based on the # of points on the written exam (sum of the points on the partial tests)
- For better grade oral exam after the written one upon appointment
- You can pass the course if you have 55% of the maximal points on the exam.

The Rules... Tests (VO)

- One test end of November, one beginning of February
- The tests are partial (half material)
- You can pass via tests if the sum of your points on both tests is at least 55% of the sum of maximal points on the tests and if on each test you have at least 20% of the maximal points
- The tests and the exams consist of exercises / questions related to the material taught in class

Some advice

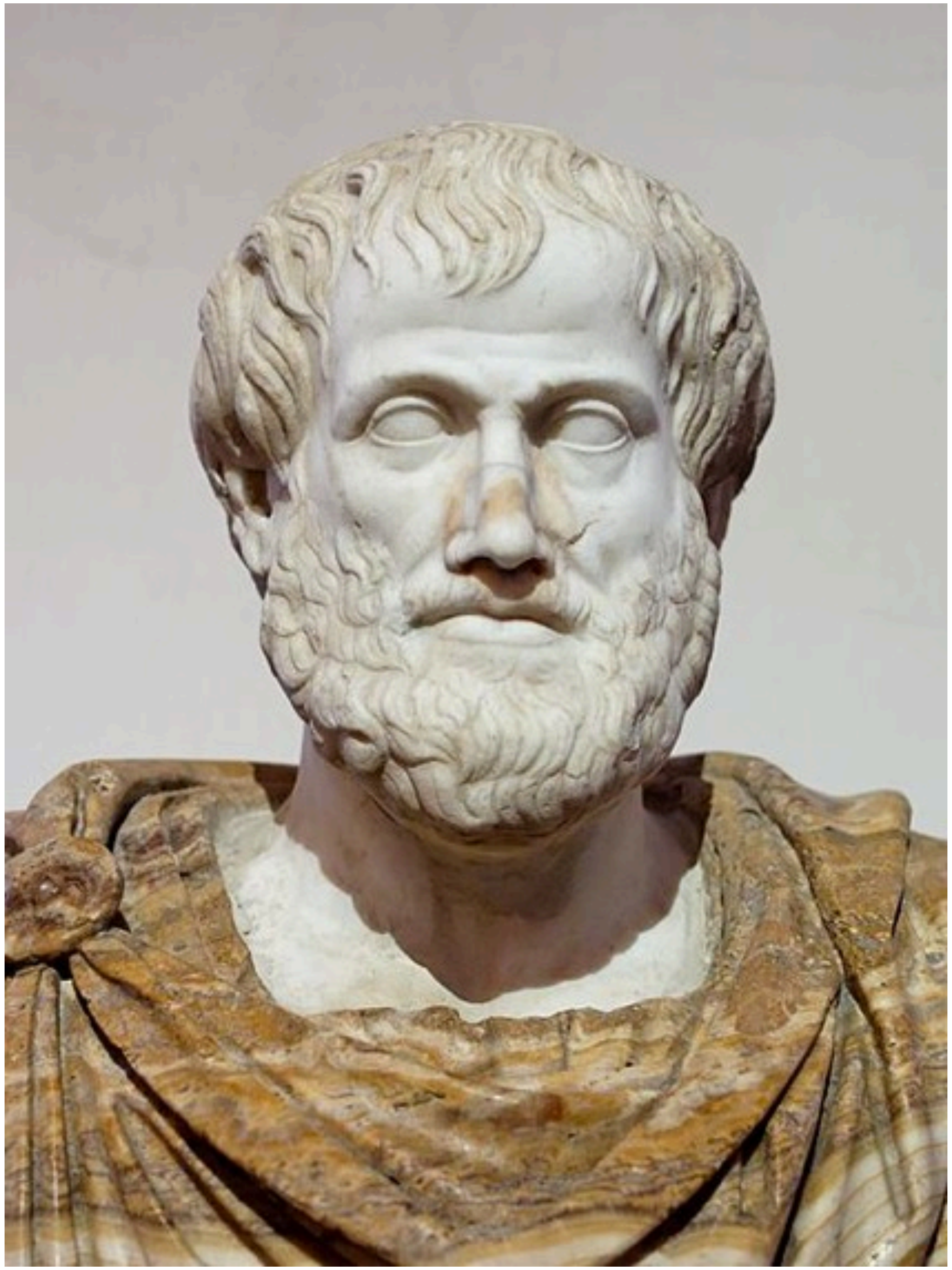
- It starts easy, but soon it gets more difficult
- There accumulates lots of material for the exam
- Best is to regularly study, practice, solve the exercises yourself!

In the beginning

Aristotle +/- 350 B.C.

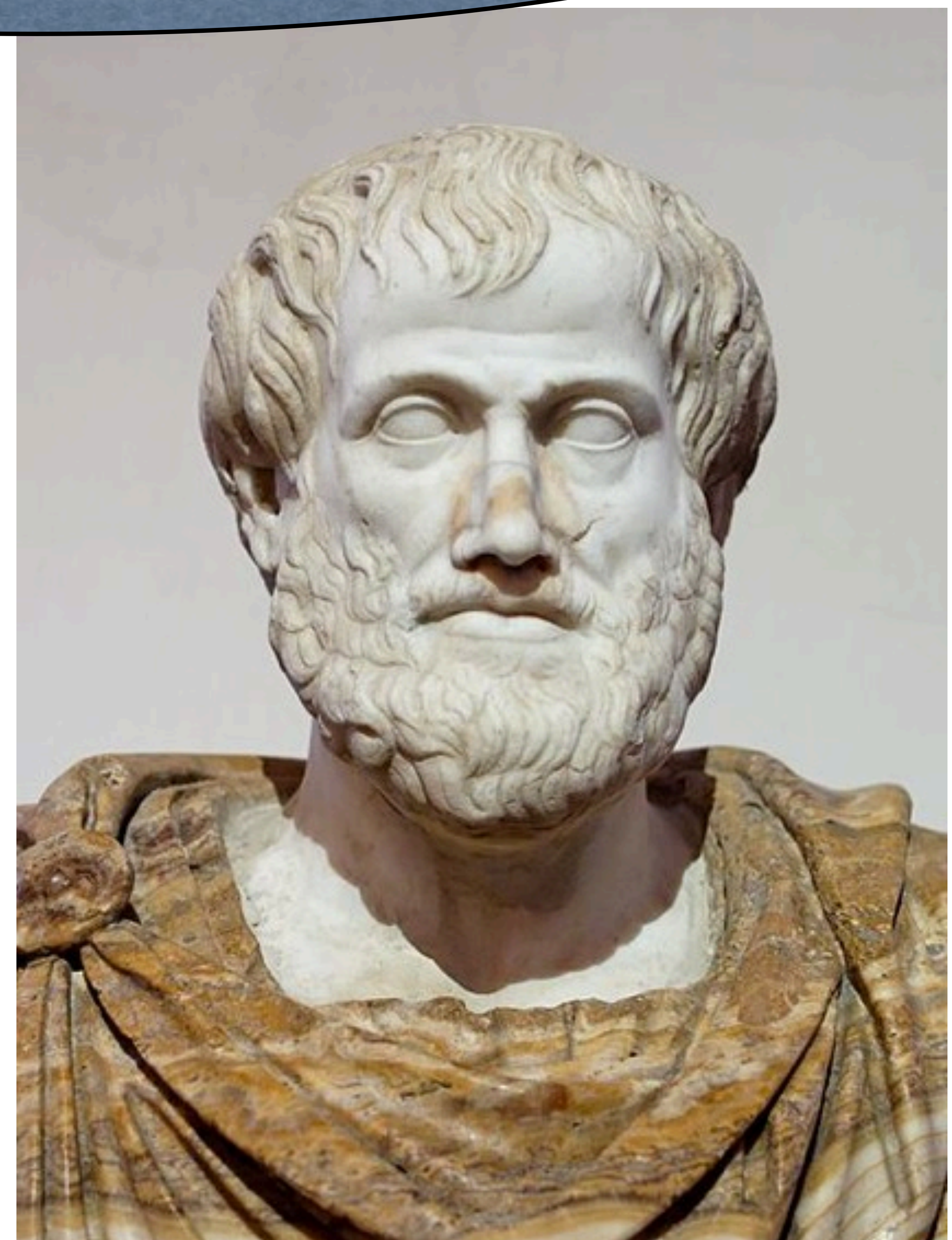
Organon

19 syllogisms



Logic = study of correct reasoning

In the beginning



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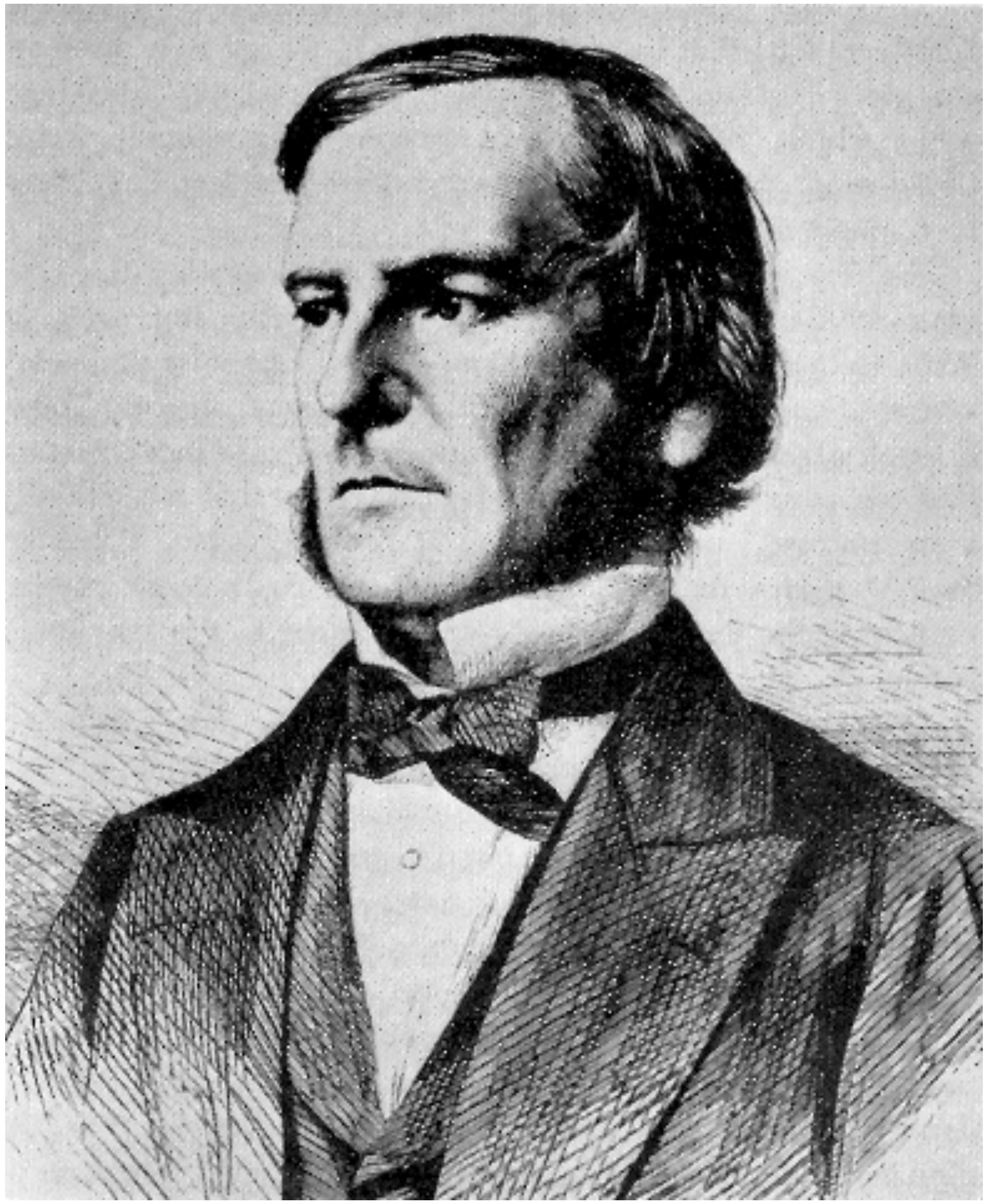
Formal Logic

Gottfried Wilhelm Leibnitz
(1646 - 1716)

Beginnings of symbolic logic

Boolean Logic



George Boole
(1815 - 1864)

Boolean logic

We will learn

- Naive Set Theory - sets, relations, mappings, numbers and structures, ordered sets
- Logical Calculations - propositional logic, predicate logic
- Logical Derivations - reasoning
- Basics of formal models - finite automata, transition systems, graphs, grammars...

We will learn

Starting today

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- Basics of formal models - finite automata, transition systems, graphs, grammars...

Why formal models/ methods?

- For better understanding of a complex system, problem, task,... models, abstractions are needed
- For rigorous precise reasoning about a complex system, problem, task

The river-crossing puzzle

- A man stands with a wolf, a goat, and a cabbage at the left bank of a river, that he wants to cross.
- The man has a boat that is large enough to carry him and another object to the other side.
- If the man leaves the wolf and the goat, or the goat and the cabbage on one side without supervision, one of them will get eaten :-(
- Is it possible to cross the river so that neither the goat nor the cabbage is eaten?

[Hopcroft et al, Kastens et al]

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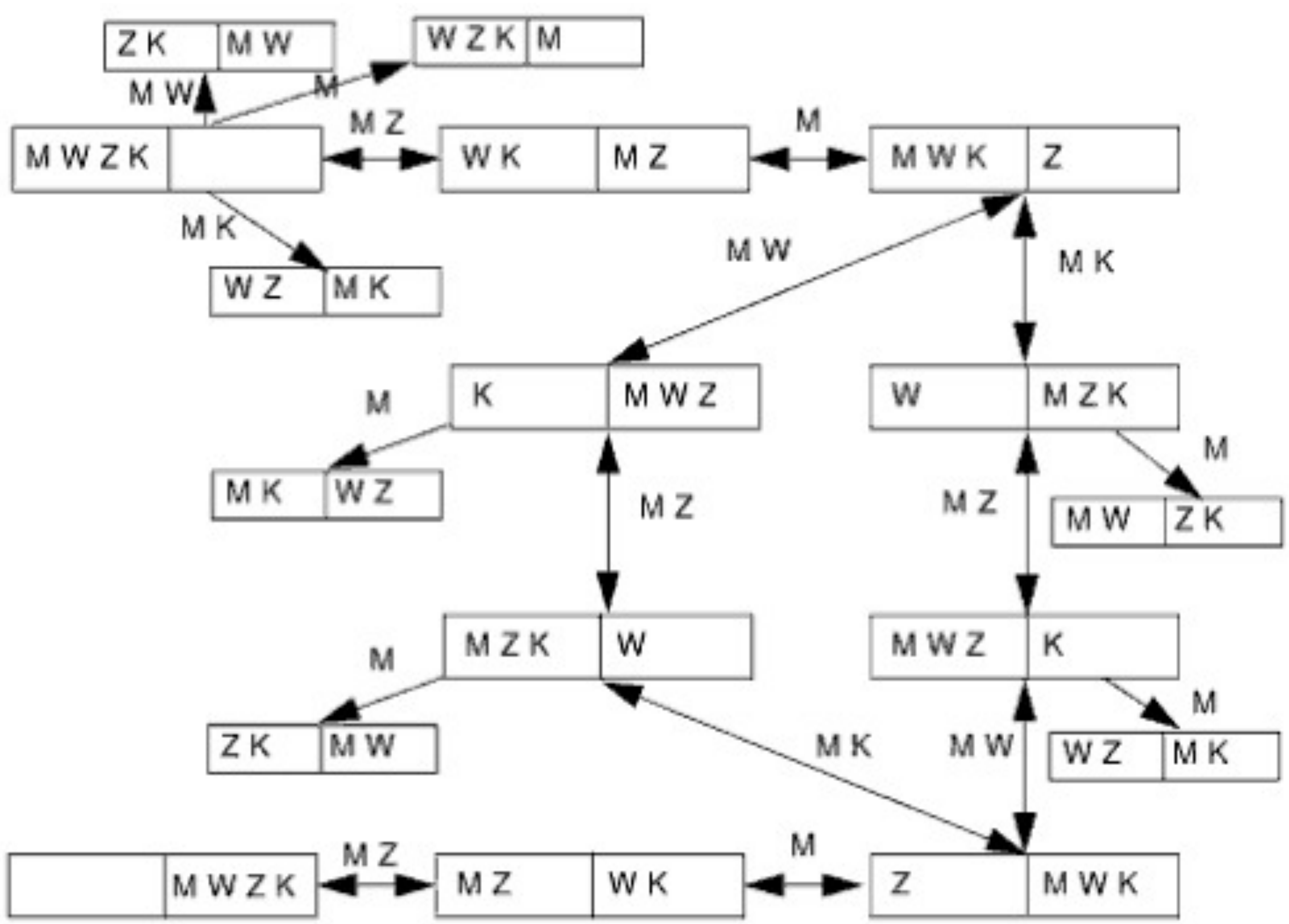

states and transitions

The river-crossing puzzle

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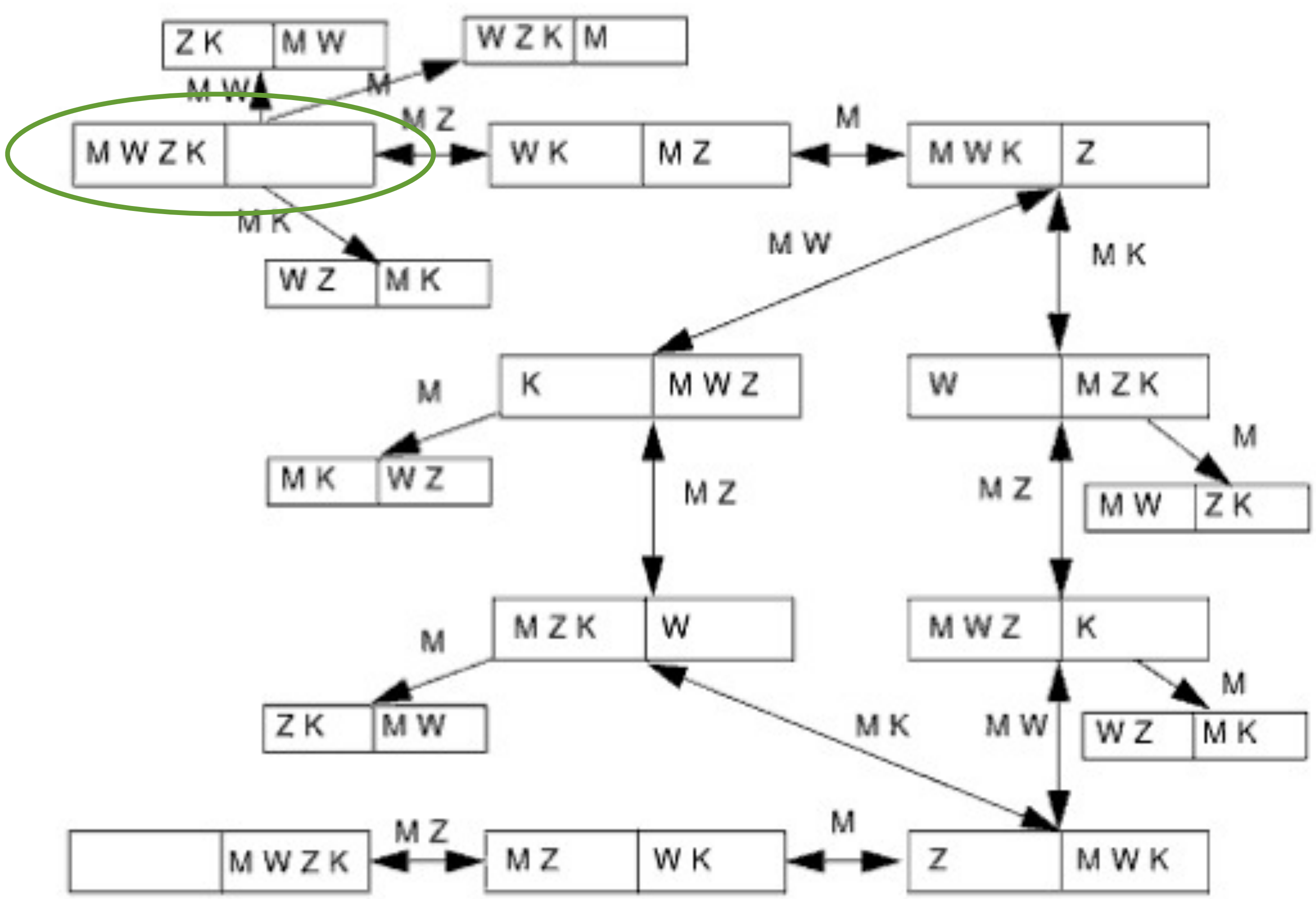

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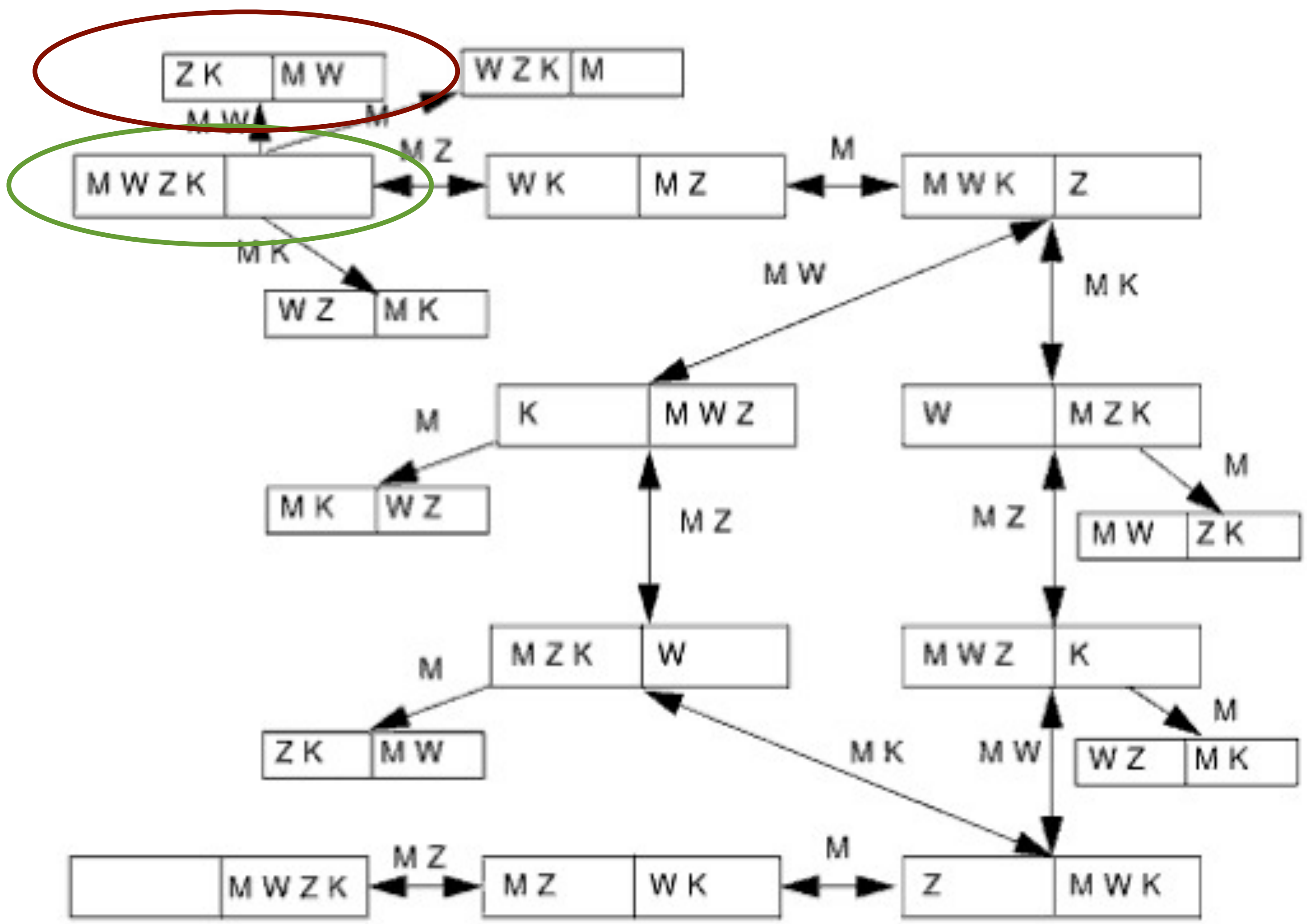

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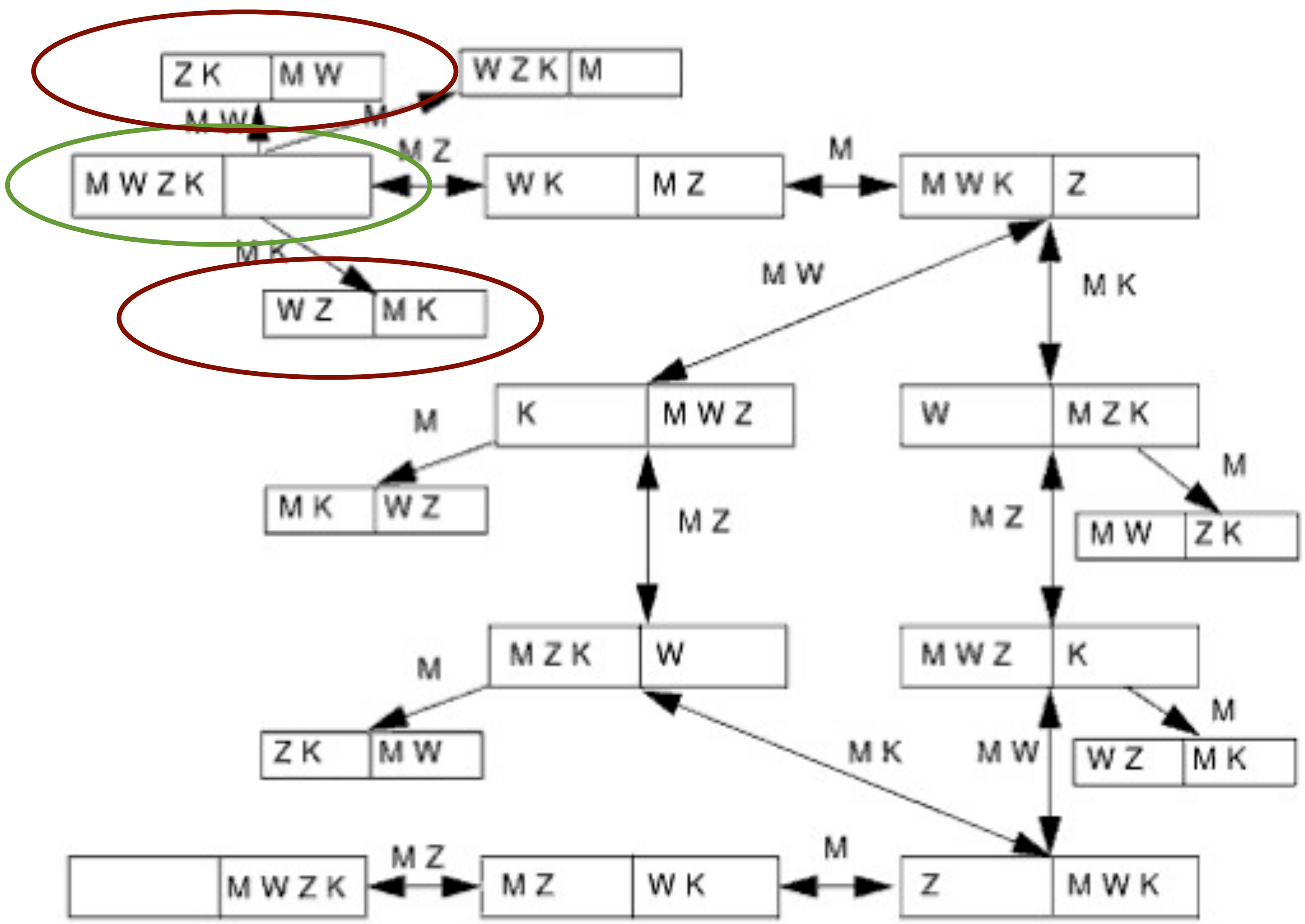

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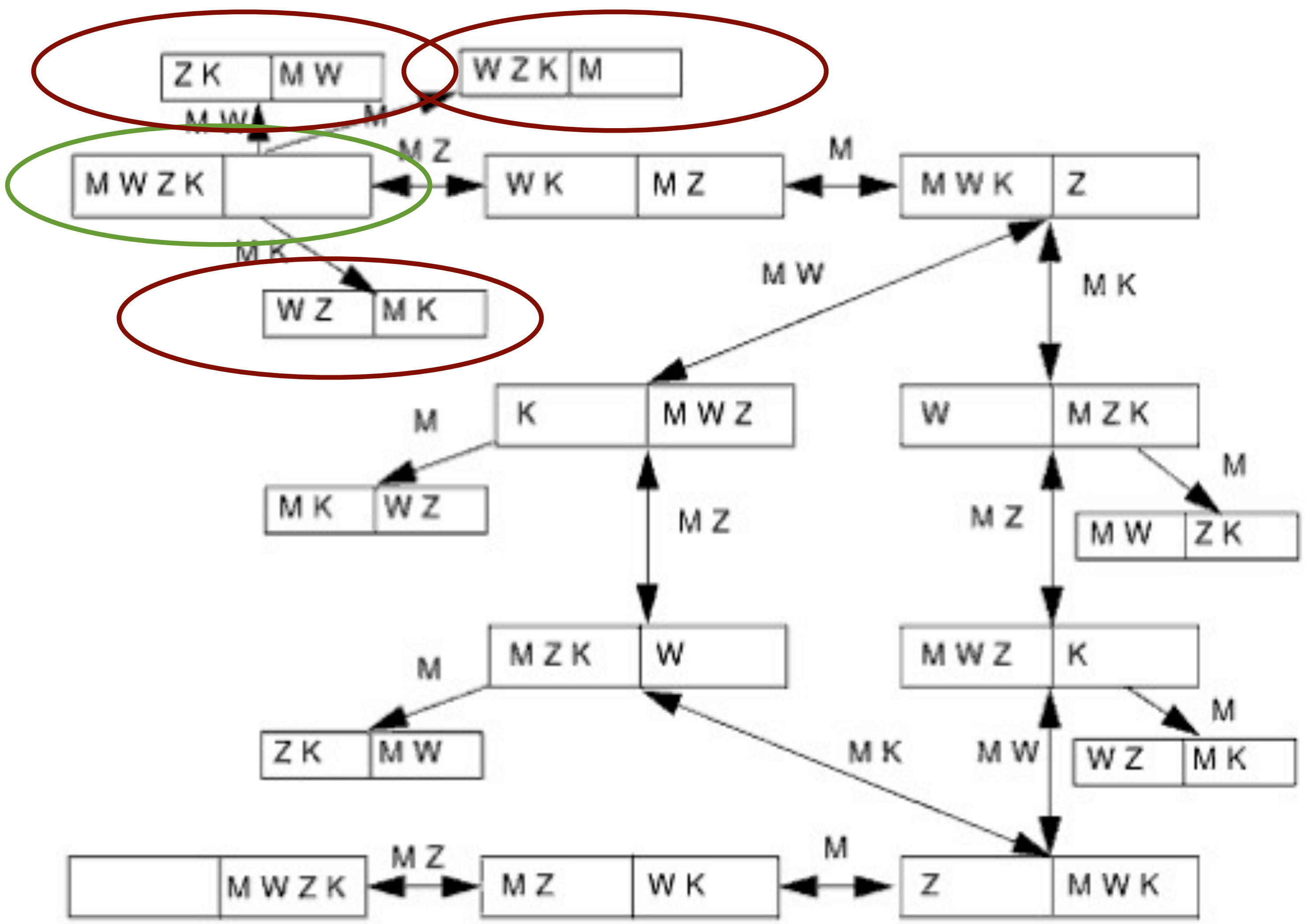

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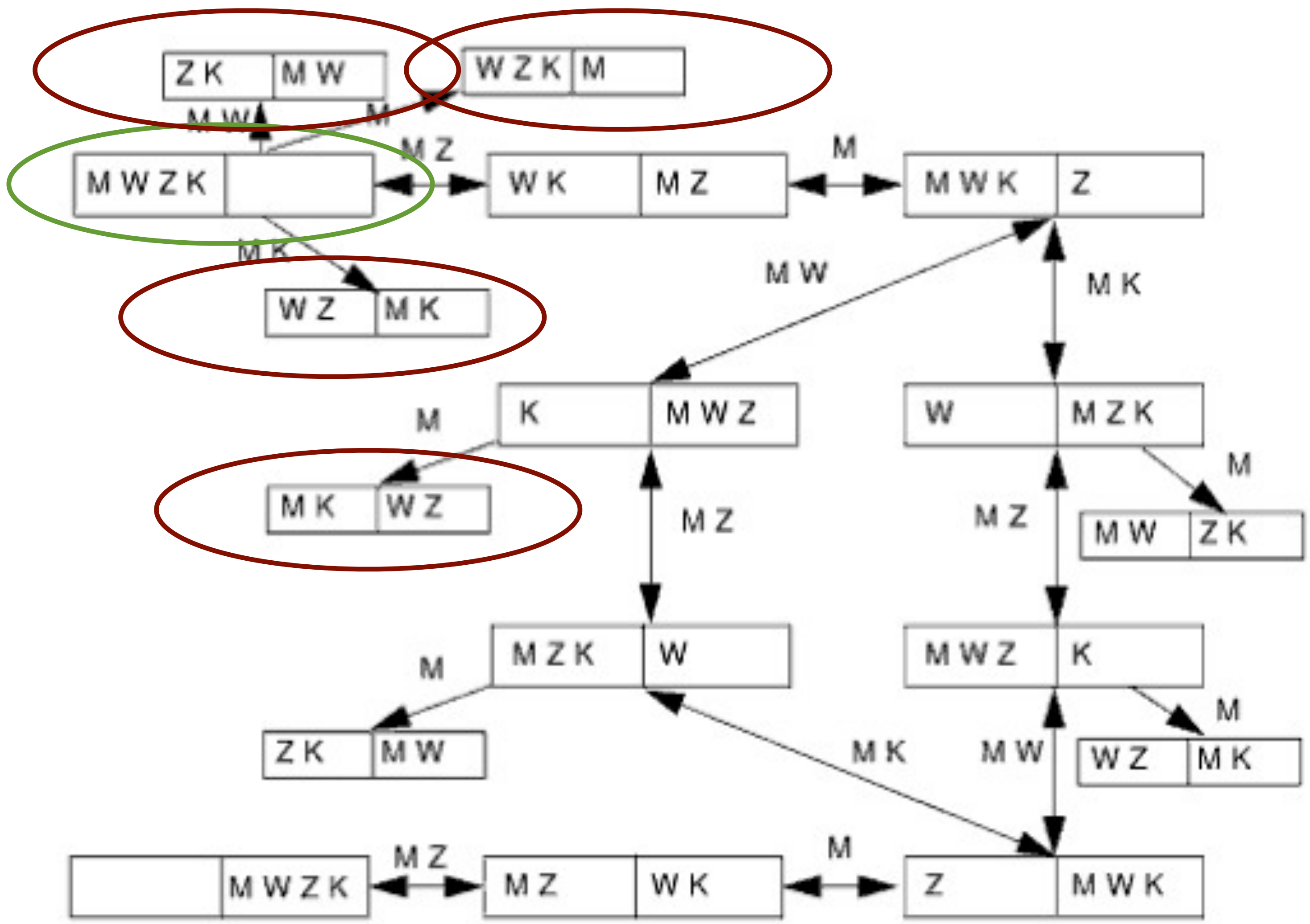

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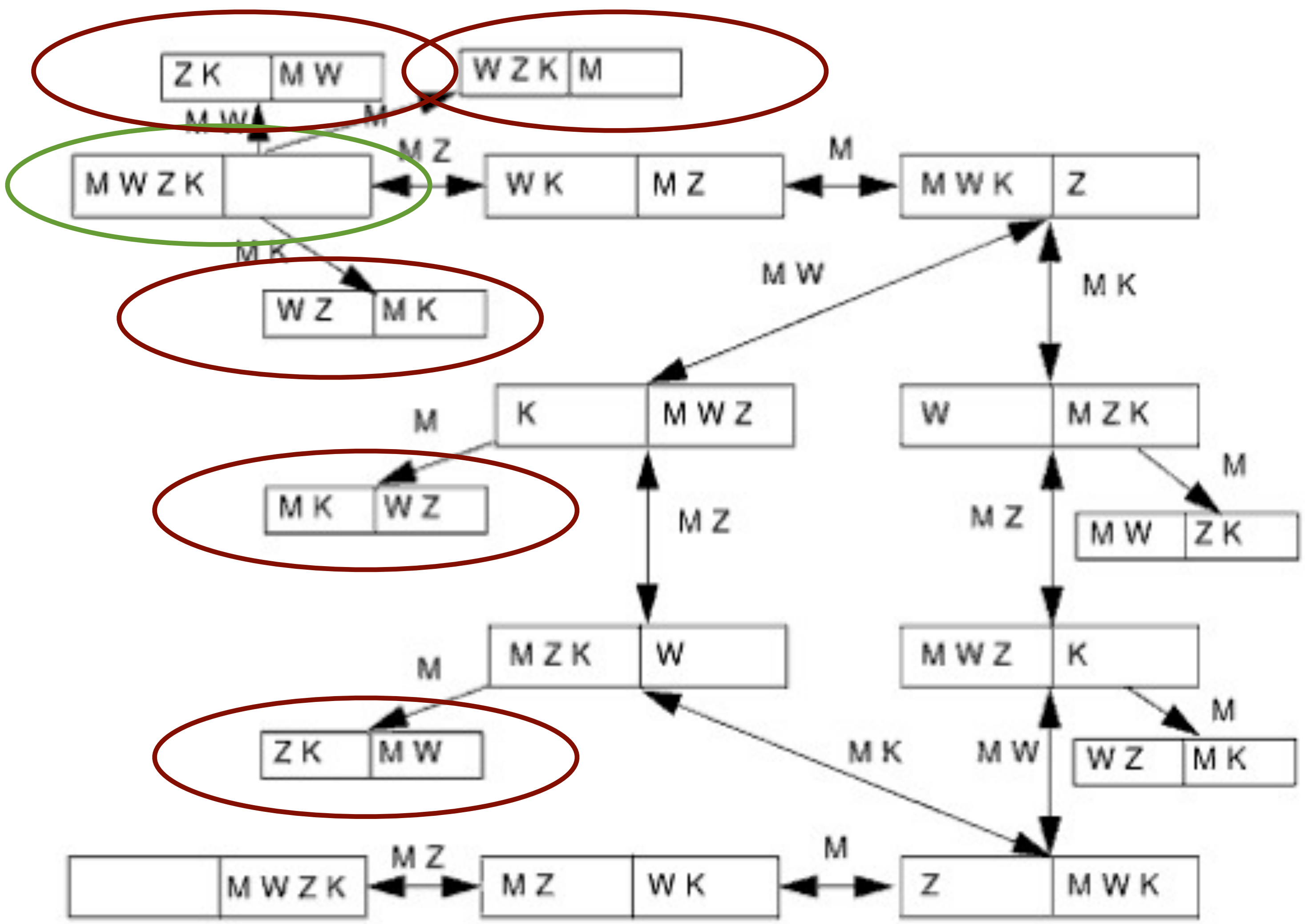

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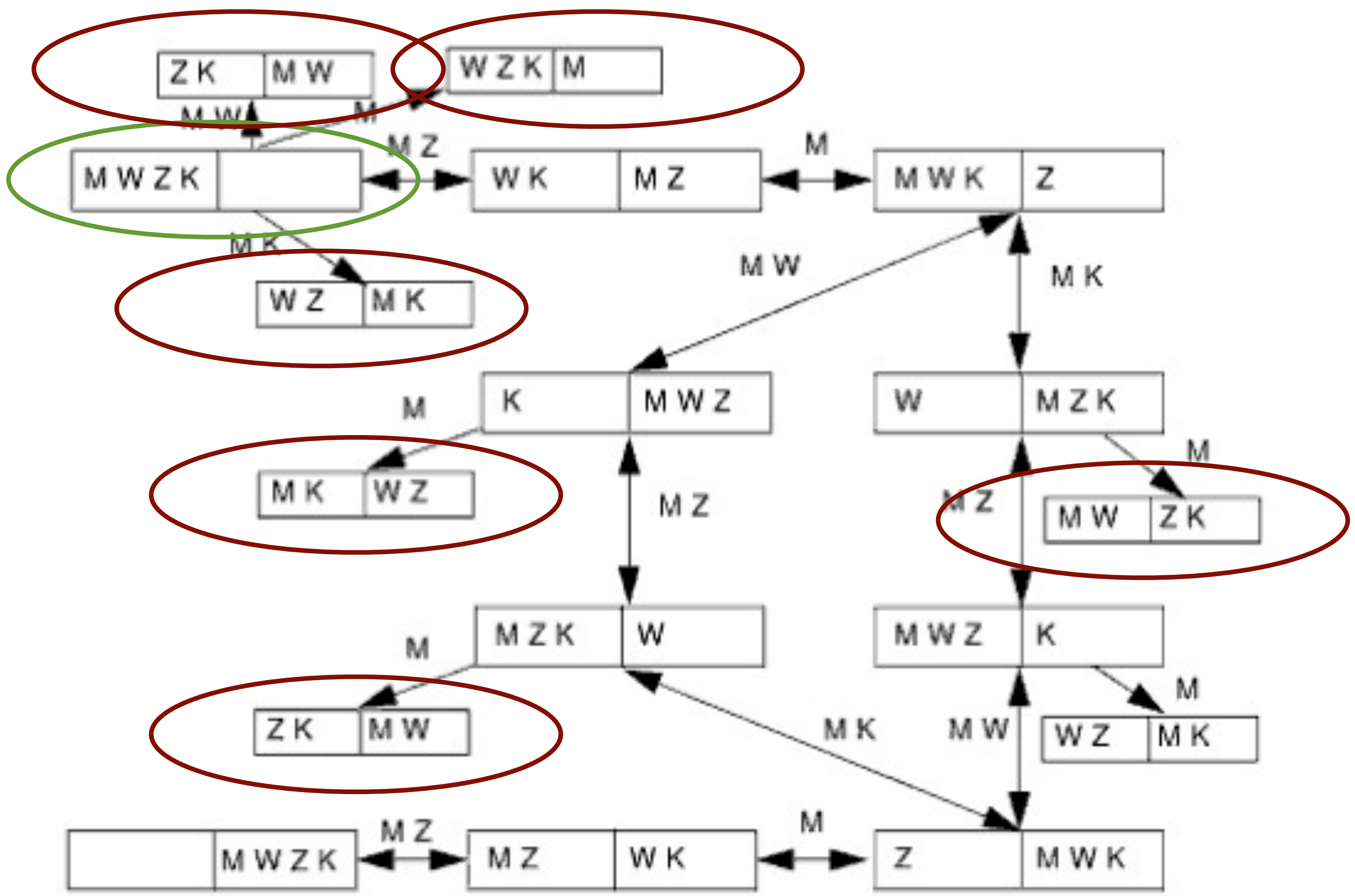

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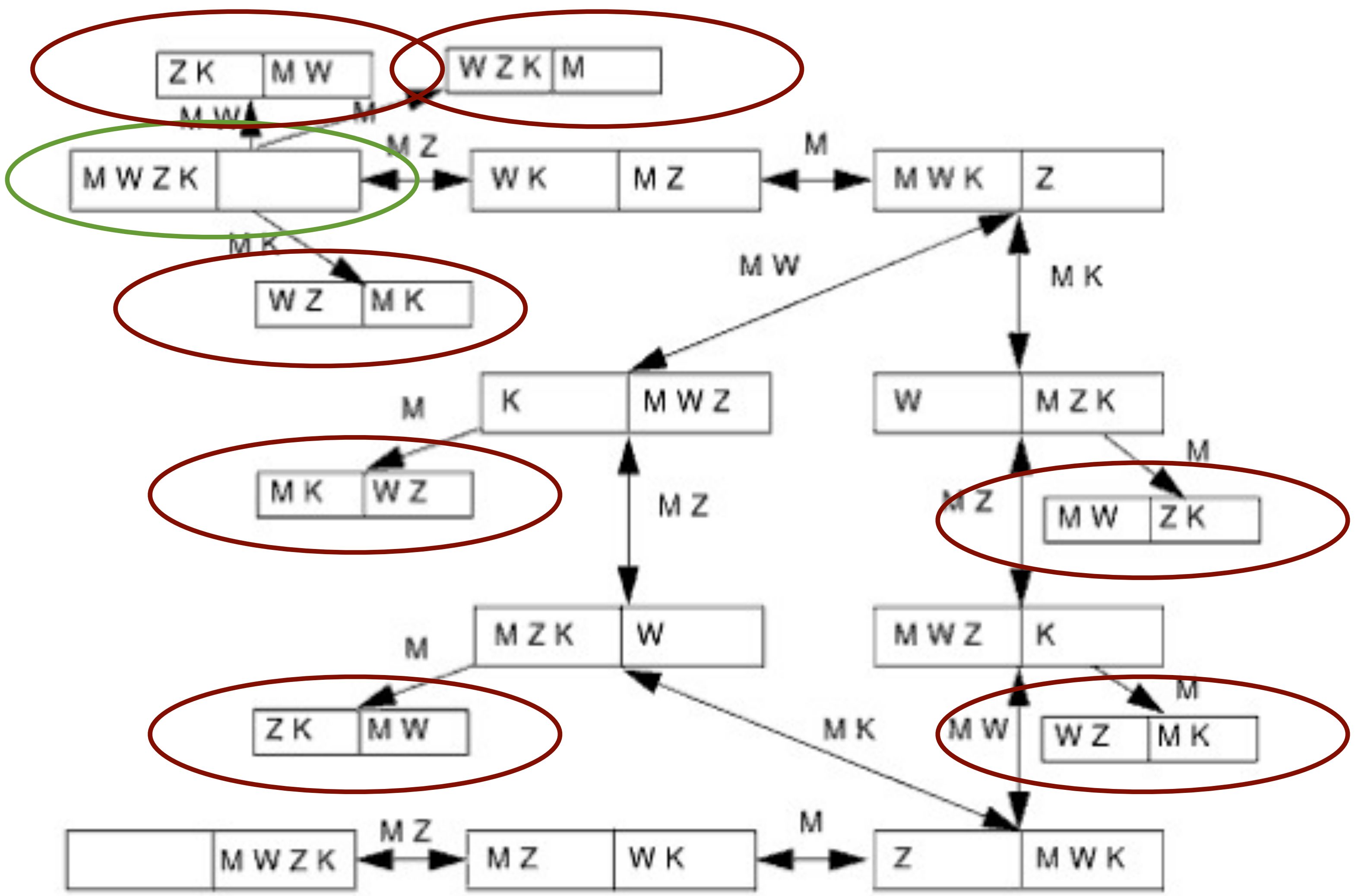

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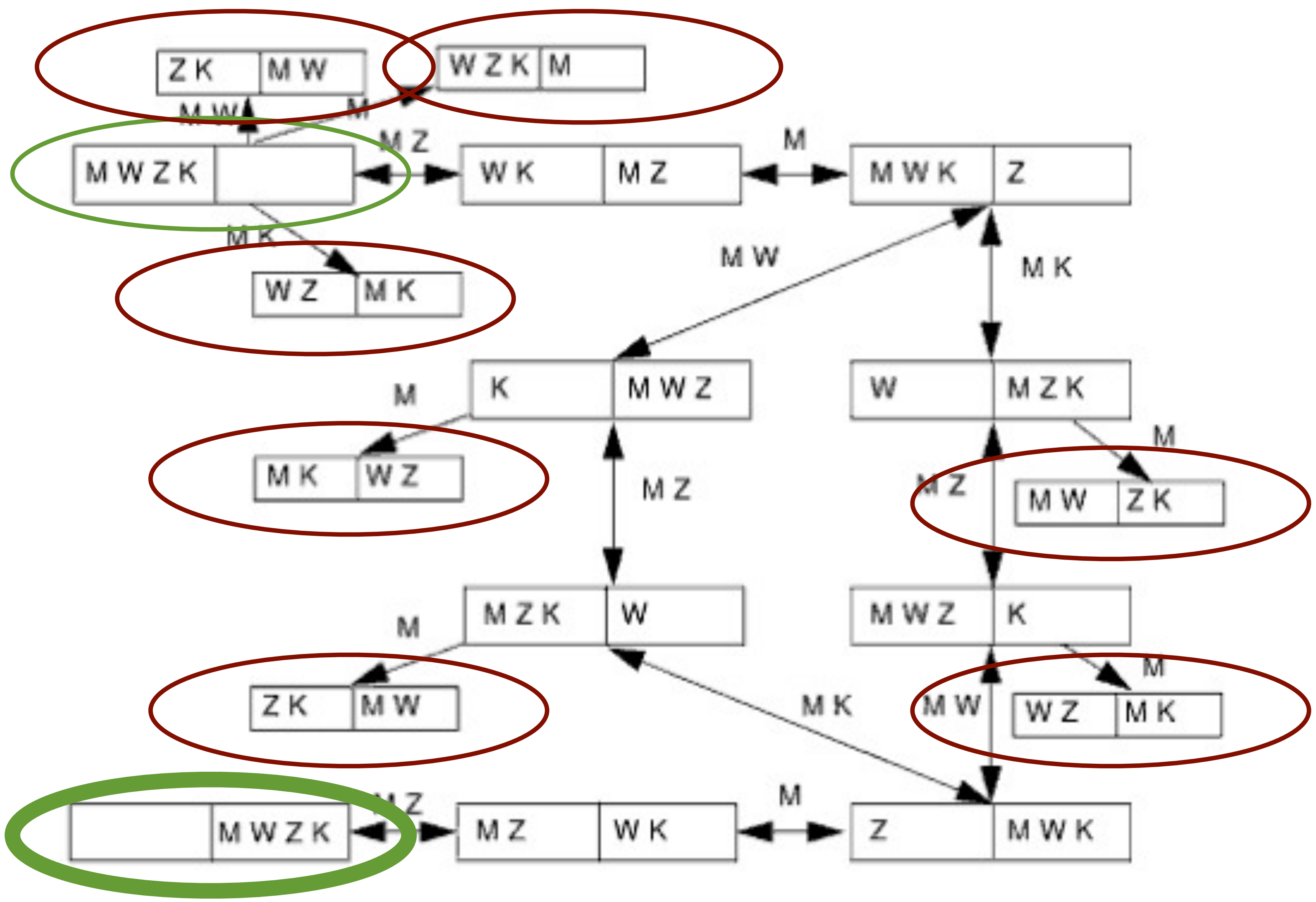
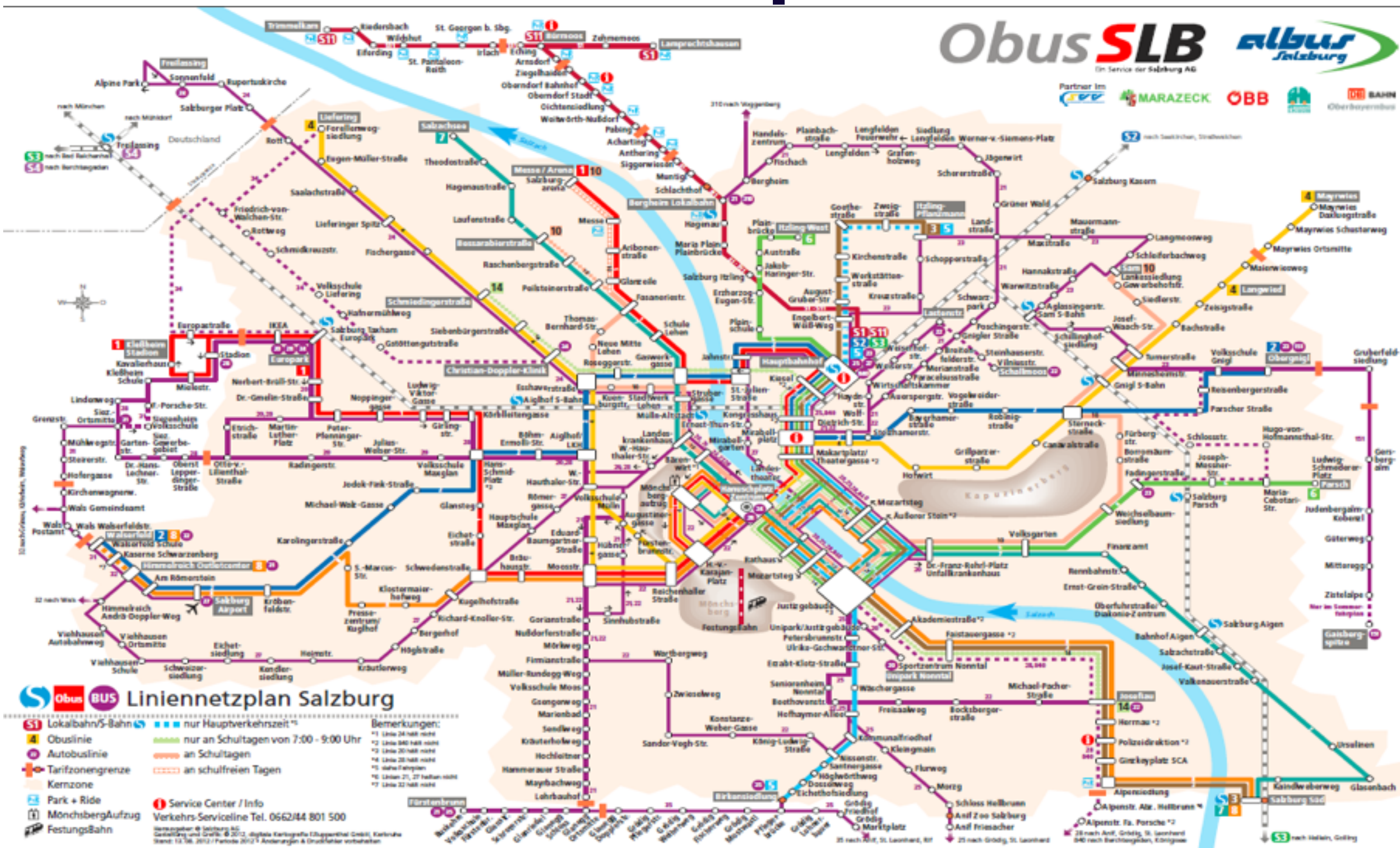


states and transitions

Another model example



Sets

- A set S is a collection of different objects, the elements of S
- We write $x \in S$ for `x is an element of S'
- A set `can' be specified by
  (1) listing its elements, e.g. S = {1,3,7,18}
  (2) specifying a property, e.g. S = {x | P(x)}
- Sets can be finite e.g. $\{\clubsuit, \heartsuit\}$ or infinite e.g. $\mathbb{N}$
- The set with no elements is the empty set, notation $\varnothing$
- The `number' of elements in a set S is the cardinality of S, notation |S|

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P is a proposition over x, which is true or false

- Sets can be finite e.g. $\{\clubsuit, \heartsuit\}$ or infinite e.g. $\mathbb{N}$

- The set with no elements is the empty set, notation $\varnothing$

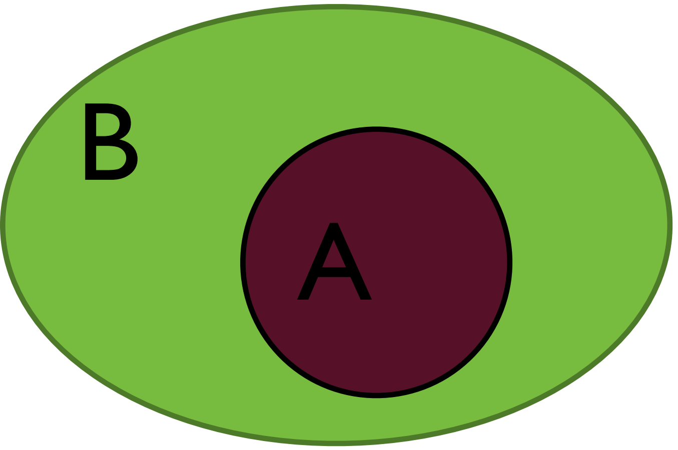
- The `number' of elements in a set S is the cardinality of S, notation $|S|$

Sets - properties

- All elements of a set are different
- The elements of a set are not ordered
- The same set can be specified in different ways, e.g. {1,2,3,4}, {2,3,1,4}, $\{i \mid i \in \mathbb{N} \text{ and } 0 < i < 5\}$

Subsets, equality

Def. $A \subseteq B$ iff all elements of A are elements of B



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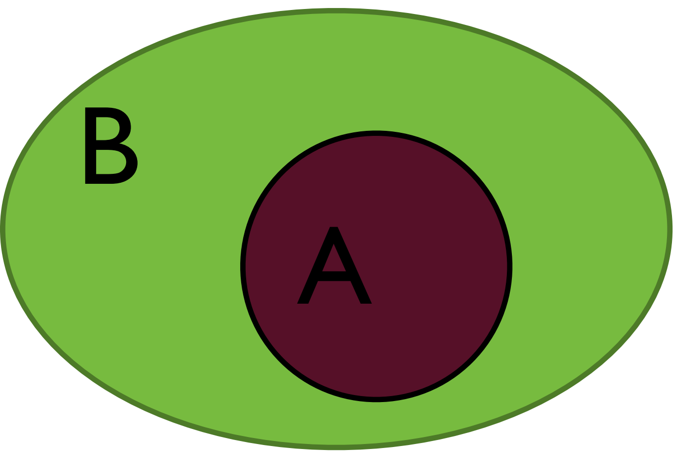
[iff for all a, if $a \in A$, then $a \in B$]



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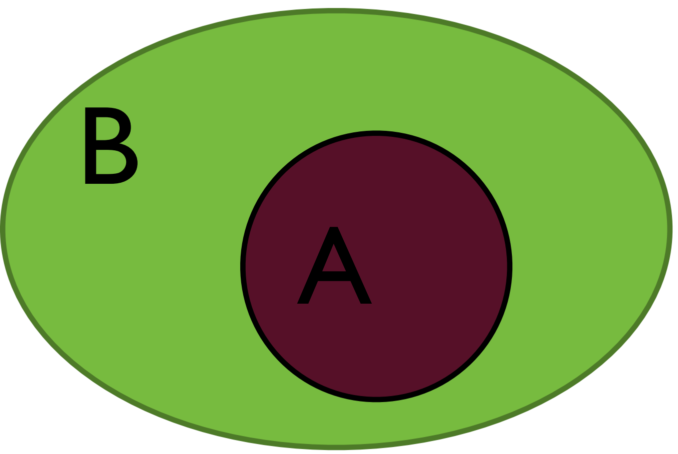
[iff $\forall a. (a \in A \Rightarrow a \in B)$]



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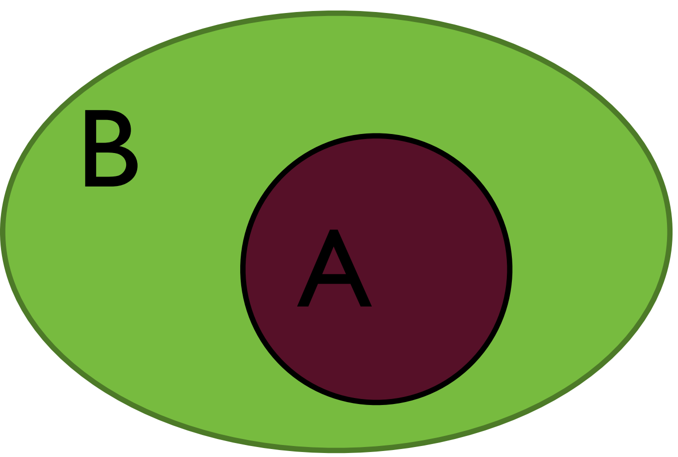

[iff $\forall a. (a \in A \Rightarrow a \in B)$]

logical formula

Subsets, equality

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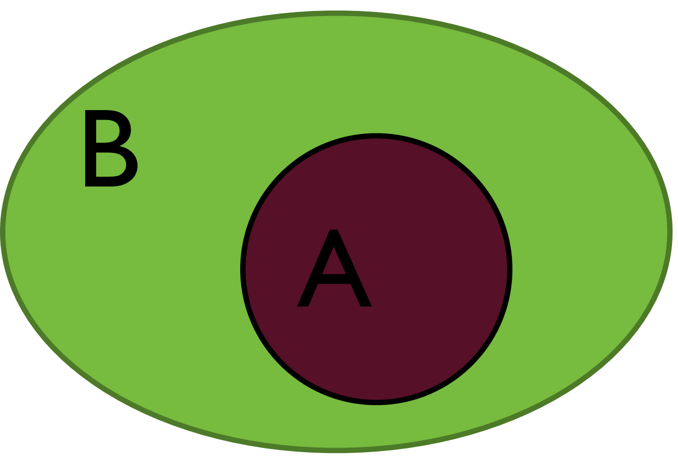

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quantifier

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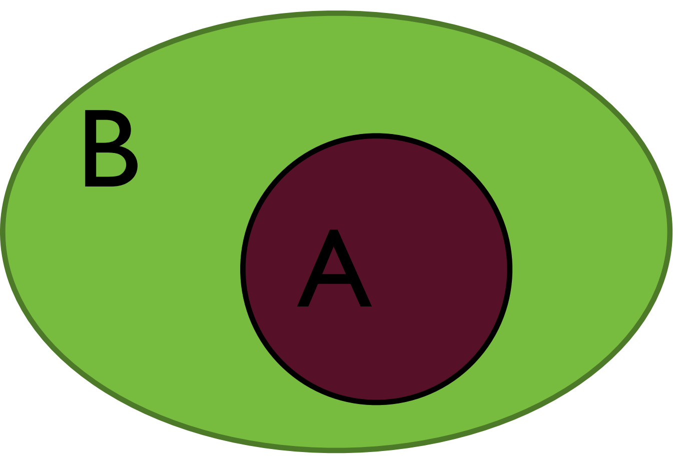

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logical connective

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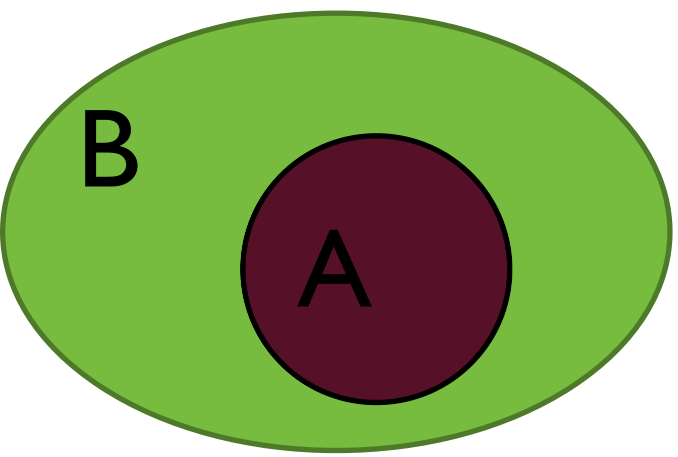

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Def. $A \subset B$ iff $A \subseteq B$ and $A \neq B$

Subsets, equality

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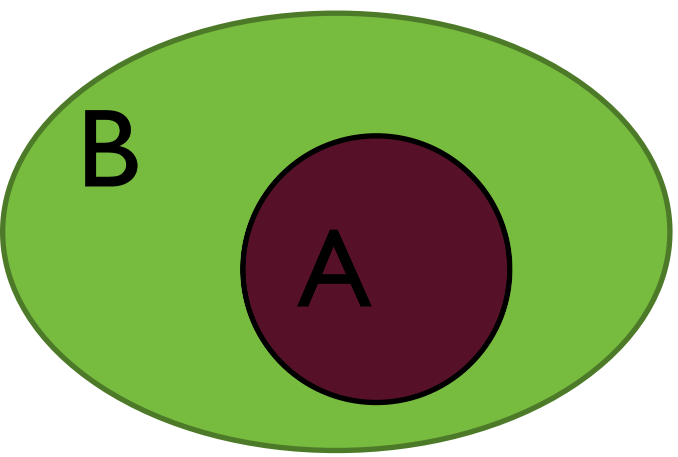
[iff for all a, if $a \in A$, then $a \in B$]



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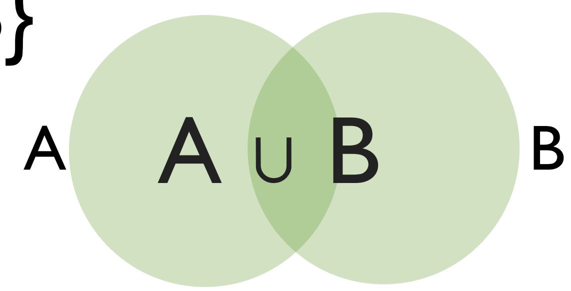
Def. $A \subset B$ iff $A \subseteq B$ and $A \neq B$

Def. $A = B$ iff $A \subseteq B$ and $B \subseteq A$

Operations on sets

Operations on sets

Union (Vereinigung) $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$



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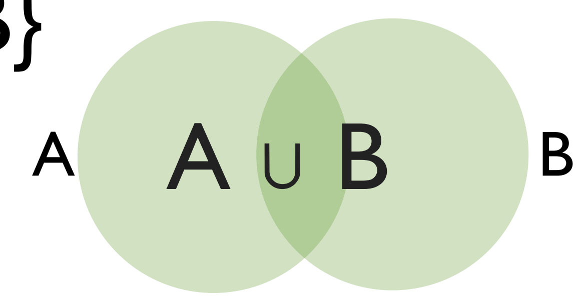
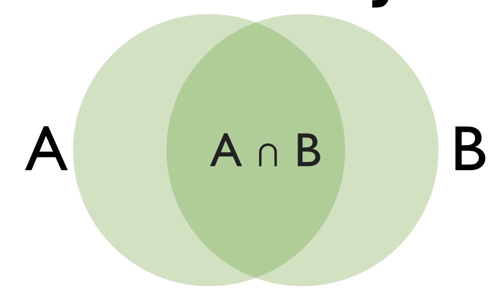


Intersection (Durchschnitt) $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$



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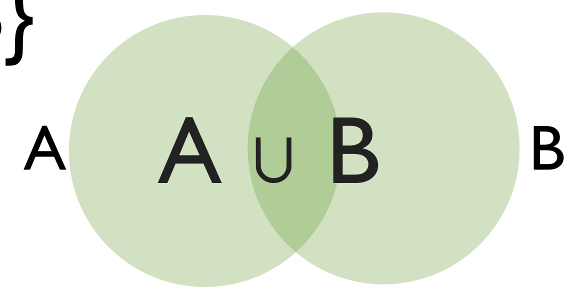
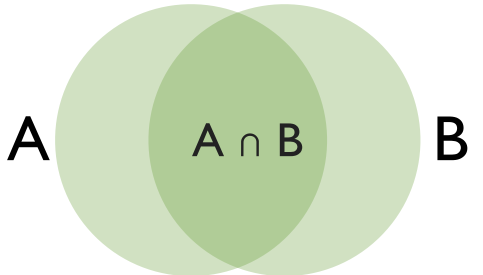


Intersection (Durchschnitt) $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

A and B are disjoint if $A \cap B = \varnothing$



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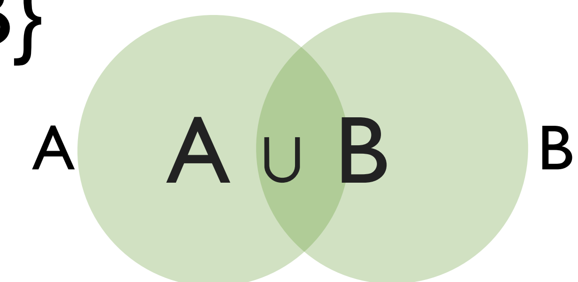

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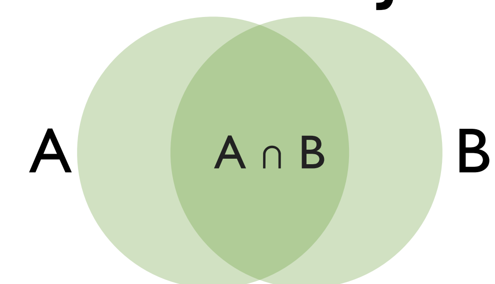
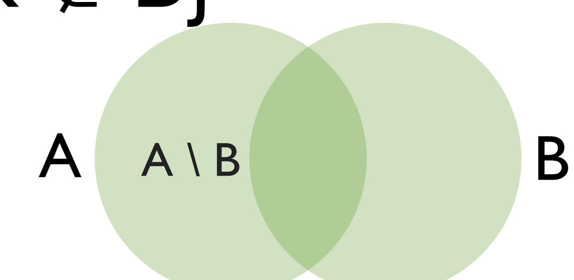


Difference (Differenz) $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$



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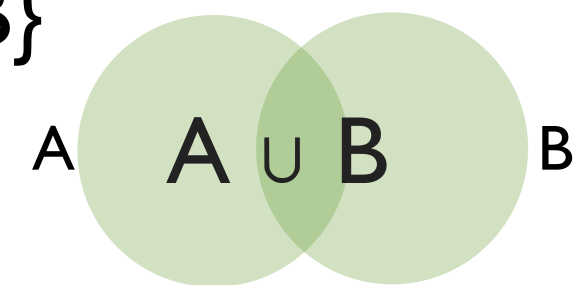

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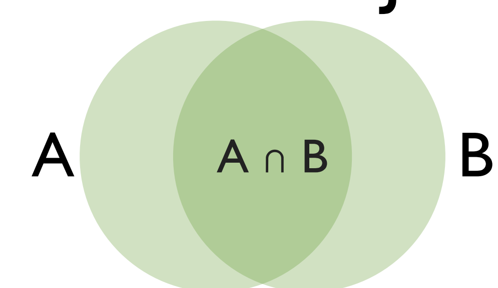

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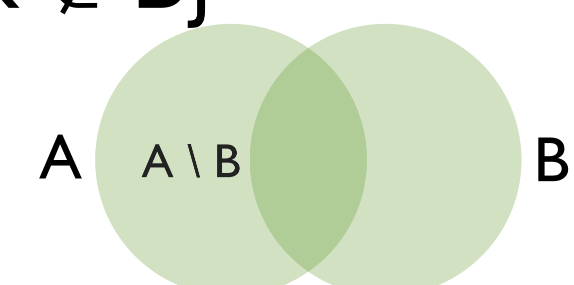

Direct product (Kartesisches Produkt)

$$A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$$

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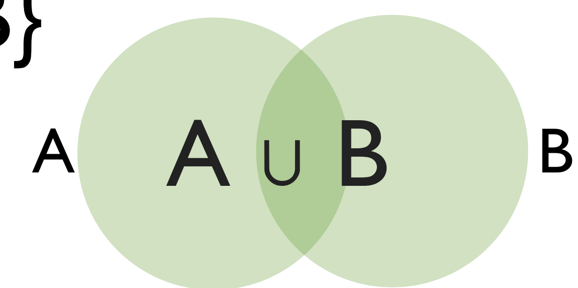

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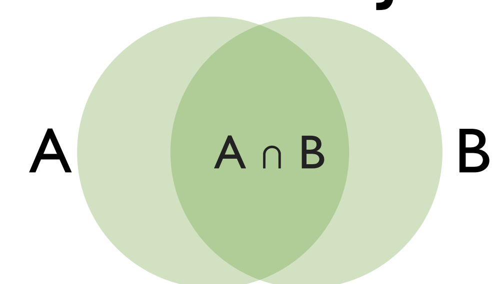

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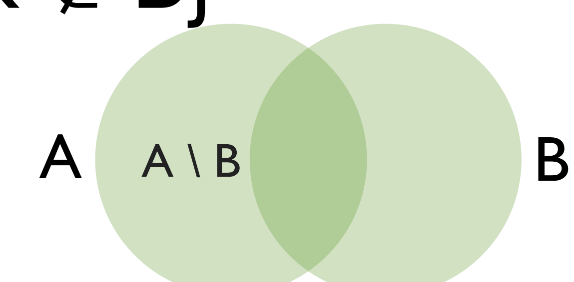

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ordered pairs

Operations on sets

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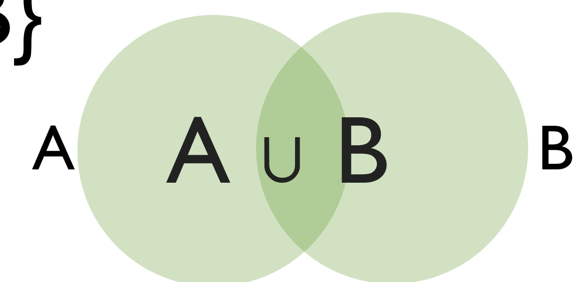

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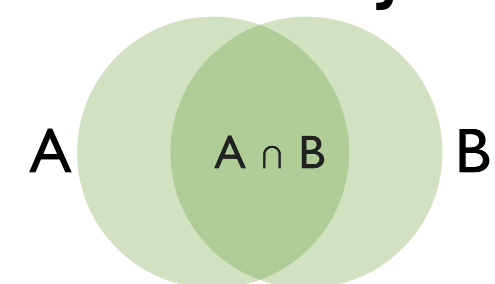

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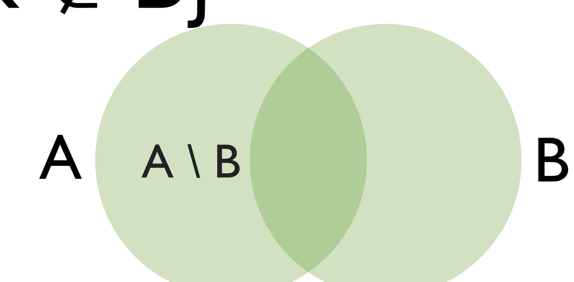

Direct product (Kartesisches Produkt)

$(A \times B) \times C \neq A \times (B \times C)$

$A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$

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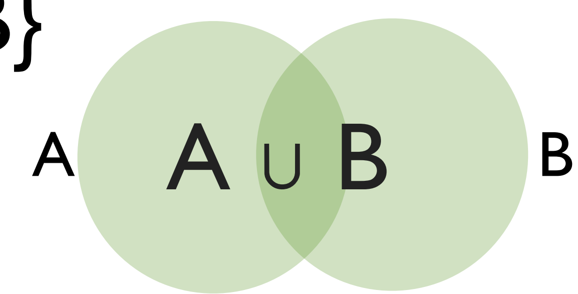

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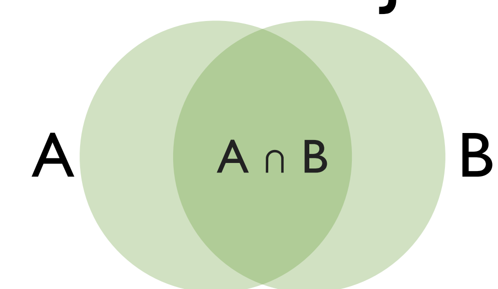
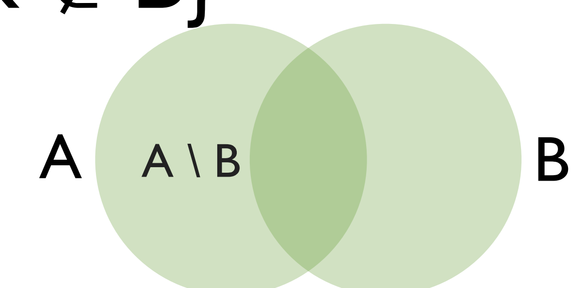
A and B are disjoint if $A \cap B = \varnothing$



Difference (Differenz) $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$



Direct product (Kartesisches Produkt)

$(A \times B) \times C \neq A \times (B \times C)$

$A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$

ordered pairs

Powerset (Potenzmenge) $\mathcal{P}(A) = \{X \mid X \subseteq A\}$

Russell's paradox

- Let P be the set of all sets that are not an element of itself
- Hence, $P = \{ x \mid x \notin x\}$
- Is $P \in P$?
- Contradiction!

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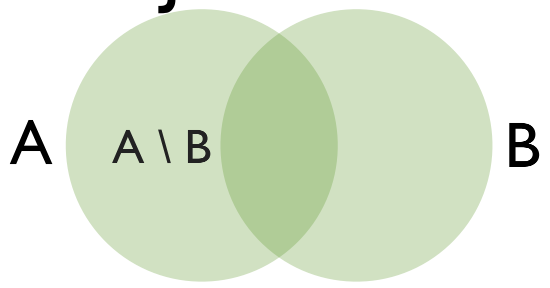
The need for a universal set U

$S = \{x \mid x \in U \text{ and } P(x)\}$

Operations on sets

Difference (Differenz) $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$



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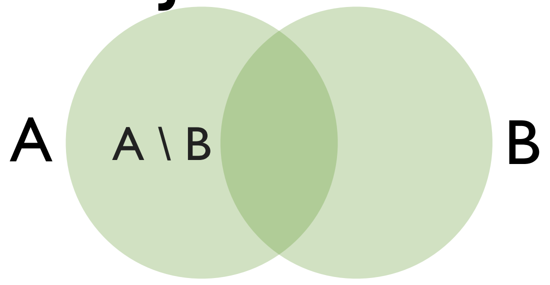
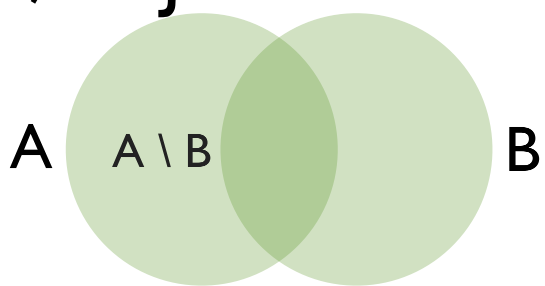


Given a universal set U

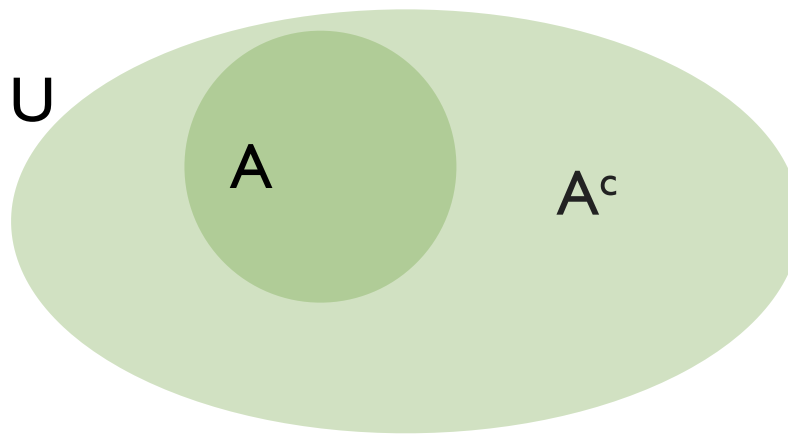
Operations on sets

Difference (Differenz) $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$



Given a universal set U

Complement (Komplement) $A^c = \{x \mid x \in U \text{ and } x \notin A\}$



Operations on sets

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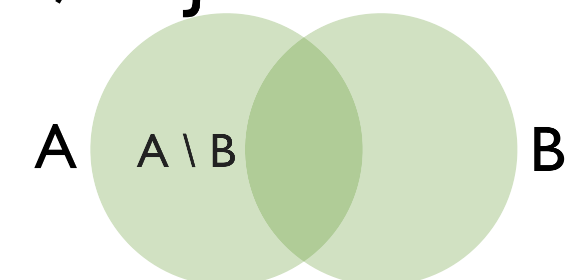

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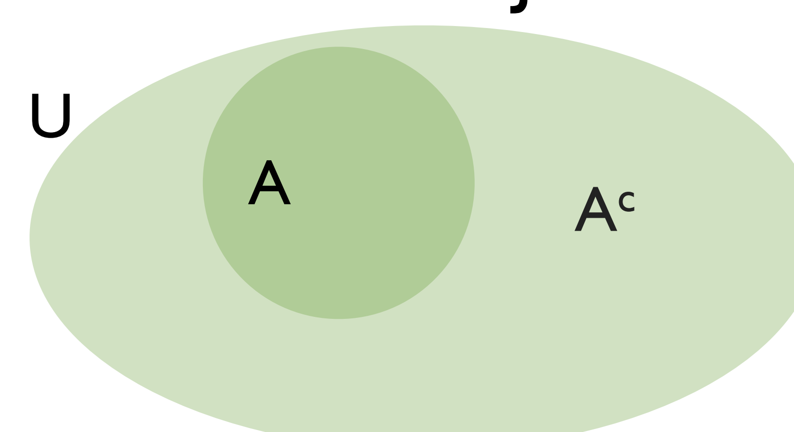

Hence $A^c = U \setminus A$

Properties of sets

1.	$\varnothing \subseteq X$
2.	If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$
3.	$X \cap Y \subseteq X$, $X \cap Y \subseteq Y$
4.	$X \subseteq X \cup Y$, $Y \subseteq X \cup Y$
5.	If $X_1 \subseteq Y_1$ and $X_2 \subseteq Y_2$, then $X_1 \cap X_2 \subseteq Y_1 \cap Y_2$
6.	If $X_1 \subseteq Y_1$ and $X_2 \subseteq Y_2$, then $X_1 \cup X_2 \subseteq Y_1 \cup Y_2$
7.	$X \cap Y = X$ iff $X \subseteq Y$
8.	$X \cap X = X$ (idempotence)
9.	$X \cup X = X$ (idempotence)
10.	$X \cap \varnothing = \varnothing$

Properties of sets

11.	$X \cup \varnothing = X$
12.	$X \cap Y = Y \cap X$ (commutativity)
13.	$X \cup Y = Y \cup X$ (commutativity)
14.	$X \cap (Y \cap Z) = (X \cap Y) \cap Z$ (associativity)
15.	$X \cup (Y \cup Z) = (X \cup Y) \cup Z$ (associativity)
16.	$X \cap (X \cup Y) = X$ (absorption)
17.	$X \cup (X \cap Y) = X$ (absorption)
18.	$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$ (distributivity)
19.	$X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$ (distributivity)
20.	$X \setminus Y \subseteq X$

Properties of sets

21.	$(X \setminus Y) \cap Y = \varnothing$
22.	$X \cup Y = X \cup (Y \setminus X)$
23.	$X \setminus X = \varnothing$
24.	$X \setminus \varnothing = X$
25.	$\varnothing \setminus X = \varnothing$
26.	If $X \subseteq Y$, then $X \setminus Y = \varnothing$
27.	$(X^c)^c = X$
28.	$(X \cap Y)^c = X^c \cup Y^c$ (De Morgan)
29.	$(X \cup Y)^c = X^c \cap Y^c$ (De Morgan)
30.	$X \times \varnothing = \varnothing$ $\varnothing \times X = \varnothing$
31.	$\varnothing \times X = \varnothing$
32.	If $X \subseteq Y$, then $\mathcal{P}(X) \subseteq \mathcal{P}(Y)$