Formale Systeme

Week 8

Reasoning

http://cs.uni-salzburg.at/~anas/teaching/FormaleSysteme/

Calculations vs Reasoning

Advantages:

- well-structured
- arguments present
- small steps
- both ways

Disadvantages:

- successive
- “standard” equivalences
- little guidance

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$$
\begin{aligned}
& (P \Rightarrow Q) \wedge P \Rightarrow Q \\
\overset{val}{=} \quad & \{\text{Implication twice}\} \\
& \neg((\neg P \vee Q) \wedge P) \vee Q \\
\overset{val}{=} \quad & \{\text{De Morgan}\} \\
& \neg(\neg P \vee Q) \vee \neg P \vee Q \\
\overset{val}{=} \quad & \{\text{De Morgan}\} \\
& (\neg\neg P \wedge \neg Q) \vee \neg P \vee Q \\
\overset{val}{=} \quad & \{\text{Double negation}\} \\
& (P \wedge \neg Q) \vee \neg P \vee Q \\
\overset{val}{=} \quad & \{\text{Distributivity}\} \\
& (P \vee \neg P \vee Q) \wedge (\neg Q \vee \neg P \vee Q) \\
\overset{val}{=} \quad & \{\text{Excludded middle twice}\} \\
& T \wedge T \\
\overset{val}{=} \quad & \{\text{T/F-elimination}\} \\
& T
\end{aligned}
$$

Calculations vs Reasoning

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Hence: $(P \Rightarrow Q) \wedge P \overset{val}{\models} Q$

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Calculations vs Reasoning

Advantages:

- easier
- more natural
- no longer successive
- single pattern
  (few useful instantiations)

(1) Assume: P
(2) Assume: $P \Rightarrow Q$
(3) From (1) and (2) deduce Q

Calculations vs Reasoning

Advantages:

Natural deduction

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  (few useful instantiations)

(1) Assume: P
(2) Assume: $P \Rightarrow Q$
(3) From (1) and (2) deduce Q

Reasoning

A proof by reasoning (natural deduction) is a sequence of statements which are either assumptions or conclusions (inferences) inferred from any of the previous (!) statements (premises)

Q is a correct conclusion from the premises (assumptions)

$P_1, \ldots, P_n$ if $P_1 \wedge \cdots \wedge P_n \overset{val}{\models} Q$

Example:

- From zero premises, any tautology is a correct conclusion
- From one premise any standard equivalence/weakening leads to a correct conclusion, e.g., from $\neg P \wedge \neg Q$ we can correctly conclude $\neg(P \vee Q)$

Types of assumptions/ hypotheses

- Pure hypotheses

- Generating hypotheses

- Identifying hypotheses / definitions

Types of assumptions/ hypotheses

- Pure hypotheses

  Example: assume $\neg(P \vee Q)$,
  assume $\forall_n[n \in \mathbb{N} : P]$,
  assume $\sqrt{2}$ is a natural number

- Generating hypotheses

- Identifying hypotheses / definitions

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Example: Let n be a natural number greater than 10

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Example: Let x be the square root of 222, i.e.,
define x = $\sqrt{222}$

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- Identifying hypotheses / definitions

By square root we mean the positive square root

Example: Let x be the square root of 222, i.e.,
define x = $\sqrt{222}$

Use of hypotheses

- To set up the scenery

- For a specific goal

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Fixed at the beginning, stay till the end

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Usually identifying hypotheses / definitions

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Usually identifying hypotheses / definitions

A pure hypothesis leads naturally to an implication

Pure / generating

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Fixed at the beginning, stay till the end

Usually identifying hypotheses / definitions

A pure hypothesis leads naturally to an implication

Pure / generating

- For a specific goal

A generating hypothesis leads naturally to a universally quantified conclusion