

Formale Systeme

Week 8

Reasoning

<http://cs.uni-salzburg.at/~anas/teaching/FormaleSysteme/>

Calculations vs Reasoning

Advantages:

- well-structured
- arguments present
- small steps
- both ways

Disadvantages:

- successive
- “standard” equivalences
- little guidance

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$$(P \Rightarrow Q) \wedge P \Rightarrow Q$$

$$\stackrel{val}{=} \{\text{Implication twice}\}$$

$$\neg((\neg P \vee Q) \wedge P) \vee Q$$

$$\stackrel{val}{=} \{\text{De Morgan}\}$$

$$\neg(\neg P \vee Q) \vee \neg P \vee Q$$

$$\stackrel{val}{=} \{\text{De Morgan}\}$$

$$(\neg\neg P \wedge \neg Q) \vee \neg P \vee Q$$

$$\stackrel{val}{=} \{\text{Double negation}\}$$

$$(P \wedge \neg Q) \vee \neg P \vee Q$$

$$\stackrel{val}{=} \{\text{Distributivity}\}$$

$$(P \vee \neg P \vee Q) \wedge (\neg Q \vee \neg P \vee Q)$$

$$\stackrel{val}{=} \{\text{Excluded middle twice}\}$$

$$T \wedge T$$

$$\stackrel{val}{=} \{\text{T/F-elimination}\}$$

$$T$$

Calculations vs Reasoning

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Hence:

$$(P \Rightarrow Q) \wedge P \stackrel{val}{=} Q$$

$$\begin{aligned} & (P \Rightarrow Q) \wedge P \Rightarrow Q \\ \stackrel{val}{=} & \quad \{\text{Implication twice}\} \\ & \neg((\neg P \vee Q) \wedge P) \vee Q \\ \stackrel{val}{=} & \quad \{\text{De Morgan}\} \\ & \neg(\neg P \vee Q) \vee \neg P \vee Q \\ \stackrel{val}{=} & \quad \{\text{De Morgan}\} \\ & (\neg\neg P \wedge \neg Q) \vee \neg P \vee Q \\ \stackrel{val}{=} & \quad \{\text{Double negation}\} \\ & (P \wedge \neg Q) \vee \neg P \vee Q \\ \stackrel{val}{=} & \quad \{\text{Distributivity}\} \\ & (P \vee \neg P \vee Q) \wedge (\neg Q \vee \neg P \vee Q) \\ \stackrel{val}{=} & \quad \{\text{Excluded middle twice}\} \\ & T \wedge T \\ \stackrel{val}{=} & \quad \{\text{T/F-elimination}\} \\ & T \end{aligned}$$

Disadvantages:

- successive
- “standard” equivalences
- little guidance

Calculations vs Reasoning

Advantages:

- easier
- more natural
- no longer successive
- single pattern
(few useful instantiations)

- (1) Assume: P
- (2) Assume: $P \Rightarrow Q$
- (3) From (1) and (2) deduce Q

Calculations vs Reasoning

Advantages:

Natural deduction

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- (1) Assume: P
- (2) Assume: $P \Rightarrow Q$
- (3) From (1) and (2) deduce Q

Reasoning

A proof by reasoning (natural deduction) is a sequence of statements which are either **assumptions** or **conclusions** (inferences) inferred from any of the previous (!) statements (premises)

Q is a correct conclusion from the premises (assumptions)

$$P_1, \dots, P_n \quad \text{if} \quad P_1 \wedge \dots \wedge P_n \stackrel{val}{\models} Q$$

Example:

- From zero premises, any tautology is a correct conclusion
- From one premise any standard equivalence/weakening leads to a correct conclusion, e.g., from $\neg P \wedge \neg Q$ we can correctly conclude $\neg(P \vee Q)$

Types of assumptions/ hypotheses

- Pure hypotheses
- Generating hypotheses
- Identifying hypotheses / definitions

Types of assumptions/ hypotheses

- Pure hypotheses

Example: assume $\neg(P \vee Q)$,
assume $\forall_n [n \in \mathbb{N} : P]$,
assume $\sqrt{2}$ is a natural number

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Example: Let n be a natural number greater than 10

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Example: Let x be the square root of 222, i.e.,
define $x = \sqrt{222}$

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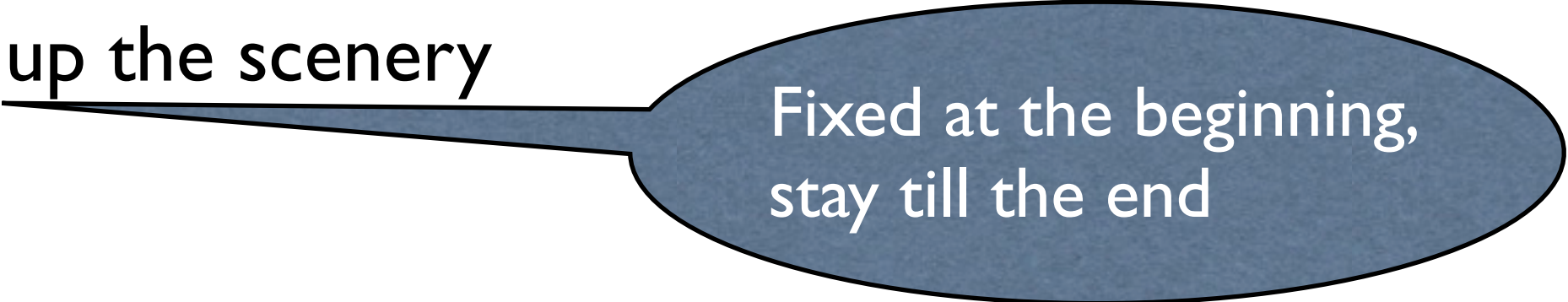
By square root we
mean the positive
square root

Use of hypotheses

- To set up the scenery
- For a specific goal

Use of hypotheses

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Fixed at the beginning,
stay till the end

- For a specific goal

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Usually identifying hypotheses / definitions

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Pure /
generating

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Fixed at the beginning,
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Usually identifying hypotheses / definitions

A pure hypothesis leads naturally to
an implication

Pure /
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Fixed at the beginning,
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Usually identifying hypotheses / definitions

A pure hypothesis leads naturally to
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Pure /
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- For a specific goal

A generating hypothesis leads naturally to
a universally quantified conclusion