Formale Systeme PS

Exercises, Week 7

Task 1. Rewrite, via a calculation, to a simpler formula

(a) $\forall_x[P : T]$

(b) $\forall_x[P : F]$

(c) $\exists_x[P : T]$

(d) $\exists_x[P : F]$

Task 2. Rewrite, via a calculation, to a simpler formula

(a) $\forall_x[P \vee Q : \neg P]$

(b) $\forall_x[P \wedge Q : \neg P]$

(c) $\exists_x[P \vee Q : \neg P]$

(d) $\exists_x[P \wedge Q : \neg P]$

Task 3. Show via a calculation that:

$$\exists_k[P : Q] \overset{val}{=} \neg\forall_k[Q : \neg P]$$

Task 4. Show via a calculation that:

$$\forall_k[P : Q \vee R] \overset{val}{=} \forall_k[P \wedge \neg Q : R]$$

Task 5. Show with a counterexample that:

$$\forall_k[P : Q] \overset{val}{\neq} \forall_k[Q : P]$$

Task 6. Show with a counterexample that:

$$\exists_k[P : Q] \wedge \exists_k[P : R] \overset{val}{\neq} \exists_k[P : Q \wedge R]$$

Task 7. Show with a calculation that

$$\exists_k[P : Q \vee R] \overset{val}{=} \exists_k[P : Q] \vee \exists_k[P : R]$$

Task 8. Show that the following formulas are tautologies.

(a) $\neg\forall_x[P \wedge Q : R] \Leftrightarrow \exists_x[P : Q \wedge \neg R]$

(b) $\neg\exists_x[\neg P \vee \neg Q : R] \Leftrightarrow \forall_x[R : P] \wedge \forall_x[R : Q]$