Formale Systeme PS

Exercises, Week 6

Task 1. The two-place predicate $A(x, y)$ on $\mathbb{R}^2$ is given by:

$$x < 3 \wedge x > y + 1.$$

(a) Check whether $A(2, 0)$, $A(2, 1)$ and $A(2, 2)$ return the truth values F or T.

(b) Draw a graph of the predicate.

(c) Give a simple formula for the one-place predicate $A(2, n)$. Draw a graph for this predicate too.

Task 2. M is the set of all people. The following predicates are given on M respectively M^2.
$Man(x) \equiv$ 'x is a man',
$Woman(x) \equiv$ 'x is a woman',
$Child(x, y) \equiv$ 'x is a child of y',
$Younger(x, y) \equiv$ 'x is younger than y'.

Give the following expressions in the form of formulas:

(a) Ana is a woman, Andreas is a man, and Andreas is younger than Ana.

(b) Michael is a man and Michael has a child.

(c) Andreas has a younger brother.

(d) All siblings of Ana are women.

Task 3. Write the following sentences as formulas with quantifiers:

(a) All natural numbers are greater than -1.

(b) All squares of real numbers are greater than or equal to 0.

(c) All real numbers that are greater than 10 are also greater than 5.

(d) If all real numbers are greater than 10, then 0 is equal to 1.

Task 4. Write the following sentences as formulas with quantifiers. D is a subset of $\mathbb{R}$.

(a) All elements of D are not equal to 0.

(b) All elements of D are greater than 10 and smaller than 25.

(c) All elements of D are greater than 10 or all elements of D are smaller than 25.

(d) Every pair of different elements of D differ by at least 1.

Task 5. Write the following sentences as formulas with quantifiers.

(a) For all natural numbers, there is a natural number which is greater than it by 1.

(b) There is no natural number which is greater than all natural numbers.

(c) The sum of two natural numbers is greater than or equal to each of these two numbers.

(d) There are two natural numbers the sum of whose squares is 58.

Task 6. Consider the following two predicates on $\mathbb{Z}^2$:

$$A(m, n) = m < n,$$

$$B(m, n) = \exists_x [x \in D : m < x < n].$$

(a) Show that $A(m, n)$ and $B(m, n)$ are equivalent if $D = \mathbb{R}$.

(b) Show that $A(m, n)$ and $B(m, n)$ are not equivalent if $D = \mathbb{Z}$.

Task 7. Check which of the following propositions are equivalent, independently of D, where D is an arbitrary subset of $\mathbb{R}$.

(a) $\exists_x [x \in D : \forall_y [y \in D : y \leq x]]$

(b) $\exists_l [l \in D : \forall_k [k \in D : l \leq k]]$

(c) $\exists_k [k \in D : \forall_m [m \in D : \neg(k < m)]]$

(c) $\forall_y [y \in D : \exists_x [x \in D : y \leq x]]$