

# Formale Systeme PS

## Exercises, Week 4

**Task 1.** Prove that:

- (a)  $P \Rightarrow Q$  is not equivalent to  $Q \Rightarrow P$
- (b)  $P \Rightarrow Q$  is not equivalent to  $\neg P \Rightarrow \neg Q$
- (c)  $P \Leftrightarrow Q \Leftrightarrow R$  is not equivalent to  $(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R)$

**Task 2.** In a mathematics book we read

$$y = x^2 + 2x + 2 \Rightarrow y = (x + 1)^2 + 1 \Rightarrow y \geq 1.$$

What is meant here? Are the arrows, implication arrows?

[Read Section 5.2.]

**Task 3.** Check if  $\Rightarrow$  respectively  $\Leftrightarrow$  are idempotent.

**Task 4.** Give shorter propositions which are equivalent to:

- (a)  $T \Leftrightarrow T$
- (b)  $T \Leftrightarrow F$
- (c)  $P \Leftrightarrow T$ .
- (d)  $P \Leftrightarrow F$ .

**Task 5.** Show that

- (a) The  $\Rightarrow$  distributes over the  $\wedge$ .
- (b) The  $\Rightarrow$  distributes over the  $\vee$ .
- (c) The  $\vee$  distributes over the  $\Leftrightarrow$ .
- (d) The  $\wedge$  does not distribute over the  $\Leftrightarrow$ .

**Task 6.** Show that Contraposition also holds for  $\Leftrightarrow$ , that is:

$$P \Leftrightarrow Q \stackrel{val}{=} \neg Q \Leftrightarrow \neg P.$$

**Task 7.** Show that:

- (a) If  $P \stackrel{val}{=} Q$  and  $Q \stackrel{val}{=} R$  and  $R \stackrel{val}{=} S$ , then  $P \stackrel{val}{=} S$ .
- (b) If  $P \stackrel{val}{=} Q$  and  $Q \stackrel{val}{\neq} R$ , then  $P \stackrel{val}{\neq} R$ .
- (c) If  $P \Leftrightarrow Q$  is a tautology and  $Q \Leftrightarrow R$  is a tautology, then  $P \Leftrightarrow R$  is a tautology.

**Task 8.** Show the following equivalences by calculating with propositions. Always state precisely: (1) which standard equivalence(s) you use, (2) whether you apply Substitution or Leibniz, or both, and (3) how you do this.

- (a)  $P \Rightarrow Q \stackrel{val}{=} (P \wedge Q) \Leftrightarrow P$
- (b)  $P \wedge (P \vee Q) \stackrel{val}{=} P$
- (c)  $P \vee (P \wedge Q) \stackrel{val}{=} P$
- (d)  $P \wedge (P \Rightarrow Q) \stackrel{val}{=} P \wedge Q$
- (e)  $P \vee (\neg P \wedge Q) \stackrel{val}{=} P \vee Q$