

Formale Systeme

Week 3

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<http://cs.uni-salzburg.at/~anas/teaching/FormaleSysteme/>

Commutativity and Associativity

Commutativity

$$P \wedge Q \stackrel{val}{=} Q \wedge P$$

$$P \vee Q \stackrel{val}{=} Q \vee P$$

$$P \Leftrightarrow Q \stackrel{val}{=} Q \Leftrightarrow P$$

Commutativity and Associativity

Commutativity

$$P \wedge Q \stackrel{val}{=} Q \wedge P$$

$$P \vee Q \stackrel{val}{=} Q \vee P$$

$$P \Leftrightarrow Q \stackrel{val}{=} Q \Leftrightarrow P$$

$$P \Rightarrow Q \stackrel{val}{\neq} Q \Rightarrow P$$

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$
0	1	1	0

Commutativity and Associativity

Commutativity

$$P \wedge Q \stackrel{val}{=} Q \wedge P$$

$$P \vee Q \stackrel{val}{=} Q \vee P$$

$$P \Leftrightarrow Q \stackrel{val}{=} Q \Leftrightarrow P$$

Associativity

$$(P \wedge Q) \wedge R \stackrel{val}{=} P \wedge (Q \wedge R)$$

$$(P \vee Q) \vee R \stackrel{val}{=} P \vee (Q \vee R)$$

$$(P \Leftrightarrow Q) \Leftrightarrow R \stackrel{val}{=} P \Leftrightarrow (Q \Leftrightarrow R)$$

Commutativity and Associativity

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$$(P \vee Q) \vee R \stackrel{val}{=} P \vee (Q \vee R)$$

$$(P \Leftrightarrow Q) \Leftrightarrow R \stackrel{val}{=} P \Leftrightarrow (Q \Leftrightarrow R)$$

$$(P \Rightarrow Q) \Rightarrow R \stackrel{val}{\neq} P \Rightarrow (Q \Rightarrow R)$$

Commutativity and Associativity

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$$(P \Leftrightarrow Q) \Leftrightarrow R \stackrel{val}{=} P \Leftrightarrow (Q \Leftrightarrow R)$$

$$(P \Rightarrow Q) \Rightarrow R \stackrel{val}{\neq} P \Rightarrow (Q \Rightarrow R)$$

P	Q	R	$(P \Rightarrow Q) \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$

Commutativity and Associativity

Commutativity

$$P \wedge Q \stackrel{val}{=} Q \wedge P$$

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$$(P \Leftrightarrow Q) \Leftrightarrow R \stackrel{val}{=} P \Leftrightarrow (Q \Leftrightarrow R)$$

$$(P \Rightarrow Q) \Rightarrow R \stackrel{val}{\neq} P \Rightarrow (Q \Rightarrow R)$$

P	Q	R	$(P \Rightarrow Q) \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$
0	1	0		

Commutativity and Associativity

Commutativity

$$P \wedge Q \stackrel{val}{=} Q \wedge P$$

$$P \vee Q \stackrel{val}{=} Q \vee P$$

$$P \Leftrightarrow Q \stackrel{val}{=} Q \Leftrightarrow P$$

Associativity

$$(P \wedge Q) \wedge R \stackrel{val}{=} P \wedge (Q \wedge R)$$

$$(P \vee Q) \vee R \stackrel{val}{=} P \vee (Q \vee R)$$

$$(P \Leftrightarrow Q) \Leftrightarrow R \stackrel{val}{=} P \Leftrightarrow (Q \Leftrightarrow R)$$

$$(P \Rightarrow Q) \Rightarrow R \stackrel{val}{\neq} P \Rightarrow (Q \Rightarrow R)$$

P	Q	R	$(P \Rightarrow Q) \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$
0	1	0	0	1

Idempotence and Double Negation

Idempotence

$$P \wedge P \stackrel{val}{=} P$$

$$P \vee P \stackrel{val}{=} P$$

$$P \Rightarrow P \stackrel{val}{\neq} P$$

$$P \Leftrightarrow P \stackrel{val}{\neq} P$$

Idempotence and Double Negation

Idempotence

$$P \wedge P \stackrel{val}{=} P$$

$$P \vee P \stackrel{val}{=} P$$

$$P \Rightarrow P \stackrel{val}{\neq} P$$

$$P \Leftrightarrow P \stackrel{val}{\neq} P$$

Double negation

$$\neg\neg P \stackrel{val}{=} P$$

T and F

Inversion

$$\neg T \stackrel{val}{=} F$$

$$\neg F \stackrel{val}{=} T$$

T and F

Inversion

$$\neg T \stackrel{val}{=} F$$

$$\neg F \stackrel{val}{=} T$$

Negation

$$\neg P \stackrel{val}{=} P \Rightarrow F$$

T and F

Inversion

$$\neg T \stackrel{val}{=} F$$

$$\neg F \stackrel{val}{=} T$$

Negation

$$\neg P \stackrel{val}{=} P \Rightarrow F$$

Contradiction

$$P \wedge \neg P \stackrel{val}{=} F$$

T and F

Inversion

$$\neg T \stackrel{val}{=} F$$

$$\neg F \stackrel{val}{=} T$$

Negation

$$\neg P \stackrel{val}{=} P \Rightarrow F$$

Contradiction

$$P \wedge \neg P \stackrel{val}{=} F$$

Excluded Middle

$$P \vee \neg P \stackrel{val}{=} T$$

T and F

Inversion

$$\neg T \stackrel{val}{=} F$$

$$\neg F \stackrel{val}{=} T$$

Negation

$$\neg P \stackrel{val}{=} P \Rightarrow F$$

Contradiction

$$P \wedge \neg P \stackrel{val}{=} F$$

Excluded Middle

$$P \vee \neg P \stackrel{val}{=} T$$

T/F - elimination

$$P \wedge T \stackrel{val}{=} P$$

$$P \wedge F \stackrel{val}{=} F$$

$$P \vee T \stackrel{val}{=} T$$

$$P \vee F \stackrel{val}{=} P$$

T and F

Inversion

$$\neg T \stackrel{val}{=} F$$

$$\neg F \stackrel{val}{=} T$$

Negation

$$\neg P \stackrel{val}{=} P \Rightarrow F$$

Contradiction

$$P \wedge \neg P \stackrel{val}{=} F$$

Excluded Middle

$$P \vee \neg P \stackrel{val}{=} T$$

T/F - elimination

$$P \wedge T \stackrel{val}{=} P$$

$$P \wedge F \stackrel{val}{=} F$$

$$P \vee T \stackrel{val}{=} T$$

$$P \vee F \stackrel{val}{=} P$$

Distributivity, De Morgan

Distributivity

$$P \wedge (Q \vee R) \stackrel{val}{=} (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \stackrel{val}{=} (P \vee Q) \wedge (P \vee R)$$

Distributivity, De Morgan

Distributivity

$$P \wedge (Q \vee R) \stackrel{val}{=} (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \stackrel{val}{=} (P \vee Q) \wedge (P \vee R)$$



De Morgan

$$\neg(P \wedge Q) \stackrel{val}{=} \neg P \vee \neg Q$$

$$\neg(P \vee Q) \stackrel{val}{=} \neg P \wedge \neg Q$$

Implication and Contraposition

Implication

$$P \Rightarrow Q \stackrel{val}{=} \neg P \vee Q$$

$$P \vee Q \stackrel{val}{=} \neg P \Rightarrow Q$$

Implication and Contraposition

Implication

$$P \Rightarrow Q \stackrel{val}{=} \neg P \vee Q$$

$$P \vee Q \stackrel{val}{=} \neg P \Rightarrow Q$$

Contraposition

$$P \Rightarrow Q \stackrel{val}{=} \neg Q \Rightarrow \neg P$$

Implication and Contraposition

Implication

$$P \Rightarrow Q \stackrel{val}{=} \neg P \vee Q$$

$$P \vee Q \stackrel{val}{=} \neg P \Rightarrow Q$$

Contraposition

$$P \Rightarrow Q \stackrel{val}{=} \neg Q \Rightarrow \neg P$$

$$P \Rightarrow Q \stackrel{val}{\neq} \neg P \Rightarrow \neg Q$$

common
mistake!

Bi-implication and Self-equivalence

Bi-implication

$$P \Leftrightarrow Q \stackrel{val}{=} (P \Rightarrow Q) \wedge (Q \Rightarrow P)$$

Bi-implication and Self-equivalence

Bi-implication

$$P \Leftrightarrow Q \stackrel{val}{=} (P \Rightarrow Q) \wedge (Q \Rightarrow P)$$

Self-equivalence

$$P \Leftrightarrow P \stackrel{val}{=}$$

Bi-implication and Self-equivalence

Bi-implication

$$P \Leftrightarrow Q \stackrel{val}{=} (P \Rightarrow Q) \wedge (Q \Rightarrow P)$$

Self-equivalence

$$P \Leftrightarrow P \stackrel{val}{=} T$$