Linearizability via Order Extension Theorems

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foundational results
for
verifying linearizability

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a glimpse into
unpublished results and
some open problems

foundational results
for
verifying linearizability

Inspiration (queue)

Queue sequential specification (axiomatic)

s is a legal queue sequence

iff

1. **s** is a legal pool sequence, and
2. enq(x) $<_{\mathbf{s}}$ enq(y) $\wedge$ deq(y) $\in$ **s** $\Rightarrow$ deq(x) $\in$ **s** $\wedge$ deq(x) $<_{\mathbf{s}}$ deq(y)

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Queue linearizability (axiomatic)

Henzinger, Sezgin, Vafeiadis CONCUR13

h is queue linearizable

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precedence order

Inspiration

As well as
Reducing Linearizability to
State Reachability
[Bouajjani, Emmi, Enea, Hamza]
ICALP15 + …

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Problems (stack)

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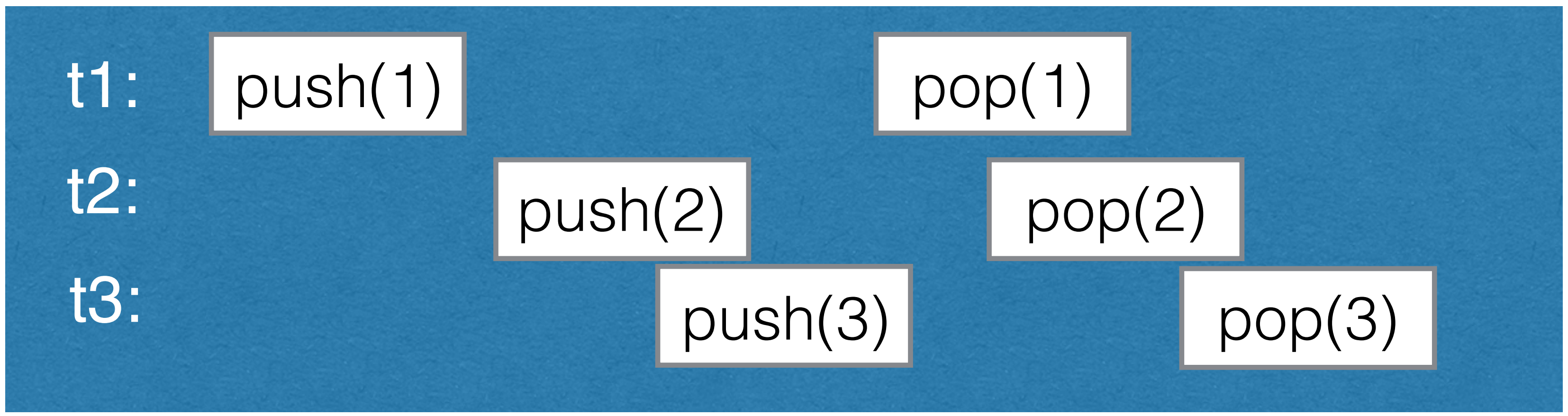

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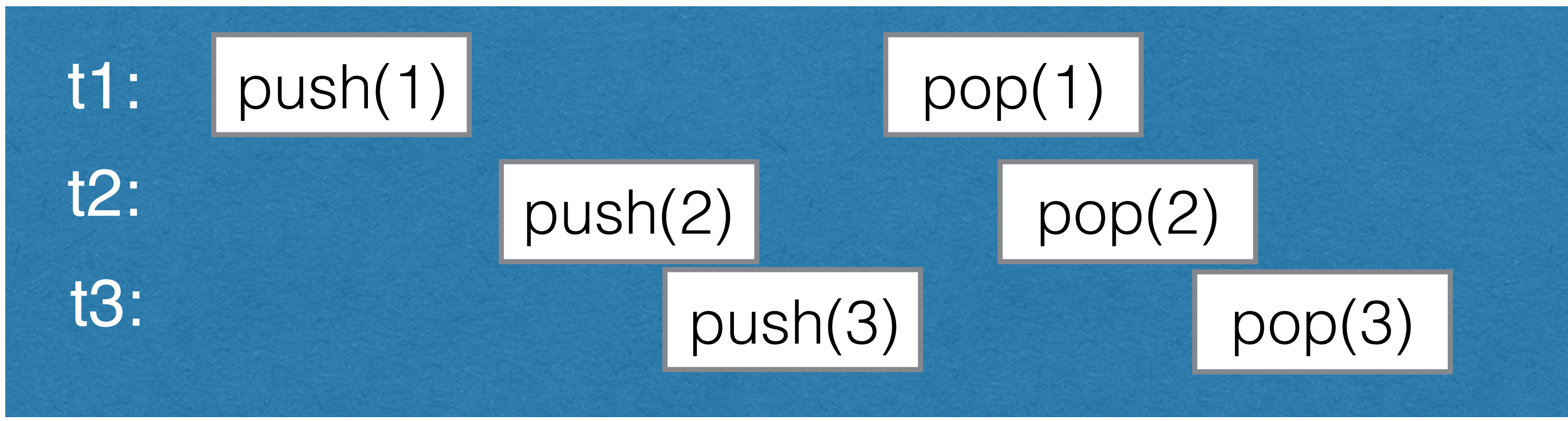

not stack linearizable

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Linearizability verification

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Data structure

- signature Σ - set of method calls including data values
- sequential specification S ⊆ Σ*, prefix closed

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identify sequences with total orders

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Sequential specification via violations

Extract a set of violations V, relations on Σ, such that **s** ∊ S iff **s** has no violations

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it is easy to find a large CV,
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we build CV iteratively from V

Pool without empty removals

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s is a legal pool (without empty removals) sequence

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CV violations
= V violations

Pool

infinite
inductive
violations

Pool sequential specification (axiomatic)

s is a legal pool (with empty removals) sequence

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2. $rem(\perp) <_{\mathbf{s}} rem(x) \Rightarrow rem(\perp) <_{\mathbf{s}} ins(x) \quad \wedge \quad ins(x) <_{\mathbf{s}} rem(\perp) \Rightarrow rem(x) <_{\mathbf{s}} rem(\perp)$

Pool

infinite inductive violations

V violations
rem(x) $<_s$ ins(x)
and
ins(x) $<_s$ rem(⊥) $<_s$ rem(x)

Pool sequential specification (axiomatic)

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Pool

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V violations
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and
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2. … …

infinitely many CV violations
$ins(x_1) <_{\mathbf{h}} rem(\perp) \wedge ins(x_2) <_{\mathbf{h}} rem(x_1) \wedge \ldots \wedge ins(x_{n+1}) <_{\mathbf{h}} rem(x_n) \wedge rem(\perp) <_{\mathbf{h}} rem(x_{n+1})$

infinite inductive violations

Queue

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infinite inductive violations

Queue

V violations
rem(x) $<_s$ ins(x)
and
ins(x) $<_s$ rem(⊥) $<_s$ rem(x)
and
enq(x) $<_s$ enq(y) ∧
deq(y) $<_s$ deq(x)

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3. enq(x) $<_s$ enq(y) ∧ deq(y) ∈ **s** ⇒ deq(x) ∈ **s** ∧ deq(x) $<_s$ deq(y)

infinite inductive violations

Queue

V violations
$rem(x) <_s ins(x)$
and
$ins(x) <_s rem(\bot) <_s rem(x)$
and
$enq(x) <_s enq(y) \wedge deq(y) <_s deq(x)$

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3. $enq(x) <_{\mathbf{s}} enq(y) \;\wedge\; deq(y) \in \mathbf{s} \quad \Rightarrow \quad deq(x) \in \mathbf{s} \;\wedge\; deq(x) <_{\mathbf{s}} deq(y)$

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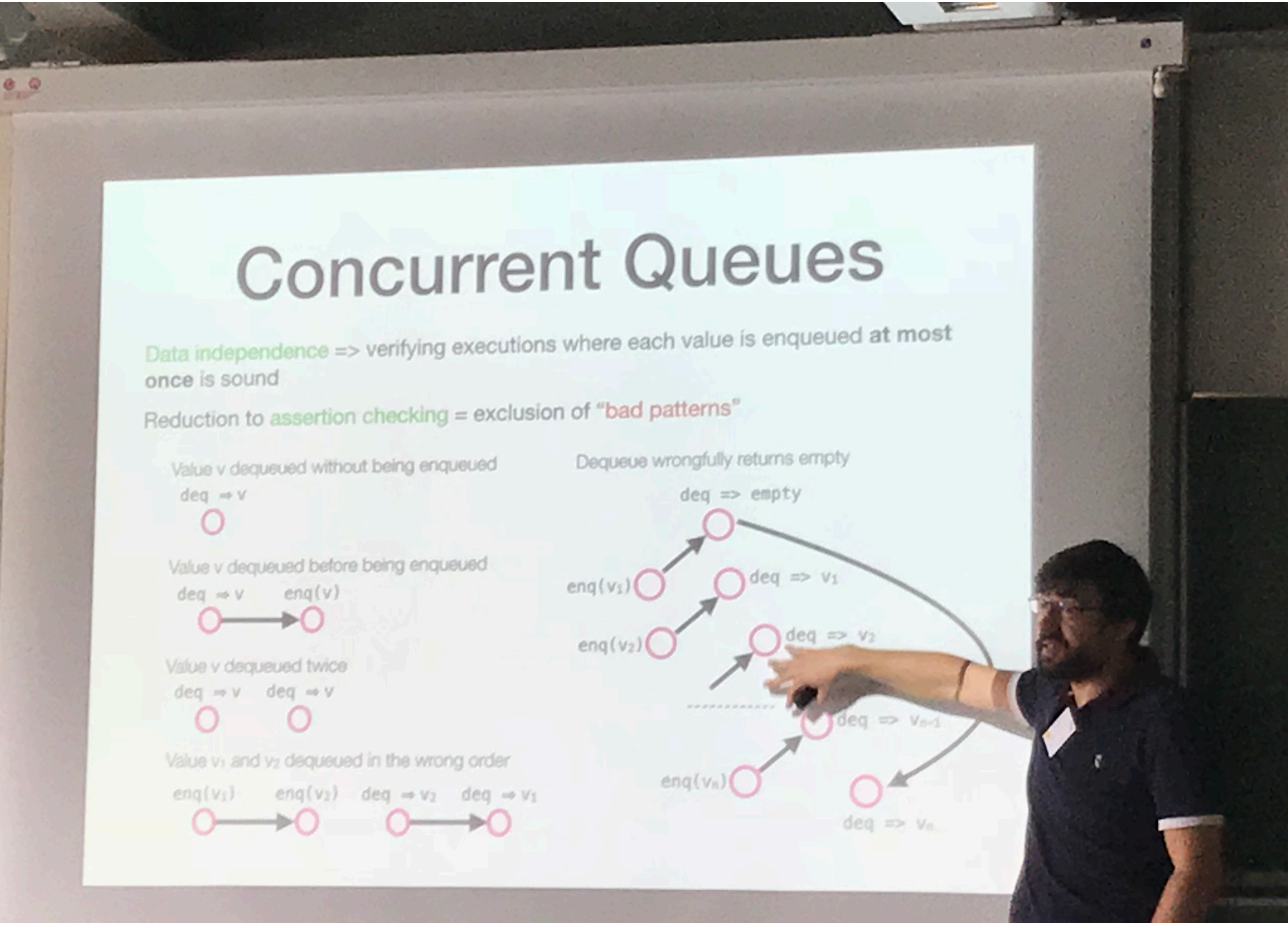
3. $enq(x) <_{\mathbf{h}} enq(y) \wedge deq(y) \in \mathbf{h} \Rightarrow deq(x) \in \mathbf{h} \wedge deq(y) \not<_{\mathbf{h}} deq(x)$


Concurrent Queues
Data independence => verifying executions where each value is enqueued at most once is sound
Reduction to assertion checking = exclusion of "bad patterns"
Value v dequeued without being enqueued
deq ⇒ v
Value v dequeued before being enqueued
deq ⇒ v
enq(v)
Value v dequeued twice
deq ⇒ v
deq ⇒ v
Value v_1 and v_2 dequeued in the wrong order
enq(v_1)
enq(v_2)
deq ⇒ v_2
deq ⇒ v_1
Dequeue wrongfully returns empty
deq => empty
enq(v_1)
deq => v_1
enq(v_2)
deq => v_2
deq => v_{n-1}
enq(v_n)
deq => v_n

It works for

- Pool without empty removals
- Queue without empty removals
- Priority queue without empty removals
- Pool
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infinite inductive violations

But not yet for Stack: infinite CV violations without clear inductive structure

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infinite inductive violations

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Exploring the space of data structures as well as new ideas for problematic cases